HEAT

COURSE OUTLINE

- ✓ **Thermometry**
- ✓ **Specific heat**
- ✓ **Gas laws**
- ✓ **Kinetic theory of gases**
- ✓ **Vapours**
- ✓ **Thermodynamics**
- ✓ **Transfer of heat**
- ✓ **Survey of energy**

Heat is a measure of energy added to or removed from a system due to presence of a hot or cold environment surrounding such a system. This energy may result into the system's temperature change or the system's temperature not changed.

This energy is equivalent to the increase in the internal energy of the system i.e. Heat = (kinetic energy + potential e energy). In this case, heat may be defined as a form of energy in transfer due to temperature differences.

In case temperature remains constant, such energy is equivalent to the increase in the system's internal energy plus work done to expand the system against external pressure i.e. Heat = (internal energy + work done against external pressure).

Temperature is the measure of hotness or coldness of the body. The extent to which the body feels hot depends on the; average kinetic energy of the individual atoms or molecules with in that body. This means that the body's kinetic energy is directly proportional to its thermal dynamic temperature.

NB: Different bodies can be at the same temperature but with different amount of heat because; Temperature of abody is a measure of its average kinetic energy only yet its heat is the sum of kinetic energy which is directly propotianal to its temperature only and molecular potential energy which is inversely propotional to its molecular separation **"r".**

Thermometry

This involves the study of thermometers as instruments used to measure temperature on the basis of certain physical thermometric properties which change with temperature and remains constant at constant temperature. **A thermometric property** is a physical quantity which varies continuously, uniformly and linearly with temperature and remains constant at constant temperature. Such properties include;

- Pressure of fixed mass of a gas at constant volume
- Electrical resistance of platinum wire
- Electromotive force of a thermocouple
- Length of liquid column (expansion of a liquid)
- Vapor pressure of vapors
- The quantity of electromagnetic radiations emitted by a hot body

NB: **A good thermometric property should;**

- Considerably vary for small changes in temperature.
- Vary over a wide range of temperature (both high and low)
- Vary linearly, uniformly and continuously with temperature
- Be accurately measurable over a wide range of temperature with a simple apparatus

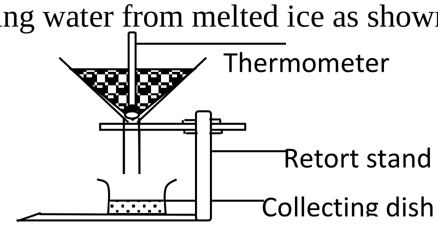
Terms used

Fixed point is defined as constant temperature at which a physical state of pure water is expected to change at 760mmHg. Fixed points are basically two i.e.0℃ and 100℃.
Lower fixed point (T_L) is the temperature of pure melting ice at 760mmHg. It is 0℃.
Upper fixed point (T_U) is the temperature of pure steam at 760mmHg. It is 100℃.
Triple point (T_{tr}) is the temperature at which pure water, pure steam and pure ice co –exist in equilibrium at 760mmHg. It is -273℃ or 0K.
Fundamental interval (F_I) is the range of the thermometer readings at the two fixed points e.g. for thermometers which give direct readings of temperature $F_I = T_U - T_L$ = (100-0)℃.

Determination of Fixed points

L.F.P

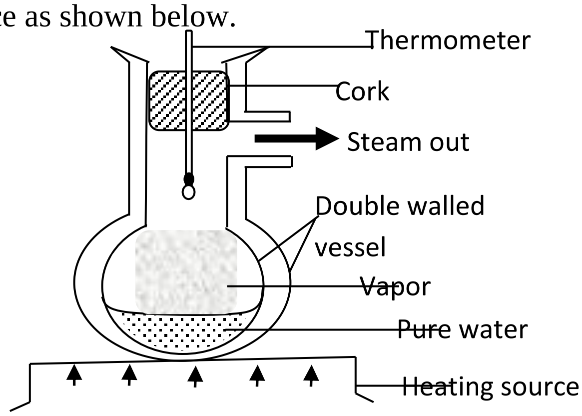
A thermometer is placed in a glass funnel kept full of pieces of pure solid ice below which is a collecting dish for collecting water from melted ice as shown below.



The mercury column/length drops steadily until when the thermometer reading remains constant at 0℃ when external pressure is760mmHg. At this state solid ice is said to be melting into liquid implying that **0℃** is the lower fixed-point of pure water.

U.F.P

A thermometer is placed inside a double walled copper vessel containing pure water below which is heating source as shown below.



The mercury thread rises steadily until when the thermometer reading remains constant at 100℃ when external pressure is760mmHg. At this state pure water is said to be vaporizing, implying that 10**0℃** is the upper fixed-point of pure water.

Note:

- All thermometers only agree at fixed points because fixed points are determined using ideal conditions i.e. (pure ice and pure steam) whose temperature only depends on standard atmospheric pressure of 760mmHg.
- Different thermometers do not agree at non fixed points because each thermometer uses a unique thermometric property which varies uniquely with temperature thus unique responses.
- Lower fixed point can not be determined using impure ice because impurities in ice lower its melting point and this would result into unique responses by different thermometers at L.F.P.
- Upper fixed point can not be determined using impure water because impurities in water raise its boiling point and this would result into unique responses by different thermometers at U.F.P.

Scale of temperature.

These are scales in which the measure of hotness or coldness of a body can be expressed .i.e. the measure of hotness or coldness of a particular body can be expressed in;

- Degrees centigrade **(℃)** forming a Celsius scale of temperature.

- Kelvin **(K)** forming a thermodynamic scale of temperature.
- Degrees Fahrenheit **(℉)** forming a Fahrenheit scale of temperature.

NB: The S.I unit for temperature is "Kelvin" with symbol "K".

Conversion of scales

If t℃ is temperature reading on the thermodynamic scale, its value on;

- Thermodynamic scale is given by the expression T= (t+273) K.
- Fahrenheit scale is given by the expression T $=\left(\frac{9}{5}t+32\right)F$.

It should however be noted that if "**T**" ℉ is temperature reading on Fahrenheit scale of temperature, its value on Celsius scale of temperature is given by the expression

t $=\left(\frac{5}{9}F-32\right)^{\circ}C$.

Establishing scales of temperature

We select a thermometric property X, and its values X_0 and X_{100} at fixed points i.e. (lower fixed point and upper fixed point) and X_{tr}at triple point are measured using a specified thermometer and recorded.

The thermometric property $X_\theta$at un known temperature θ is also measured and recorded.

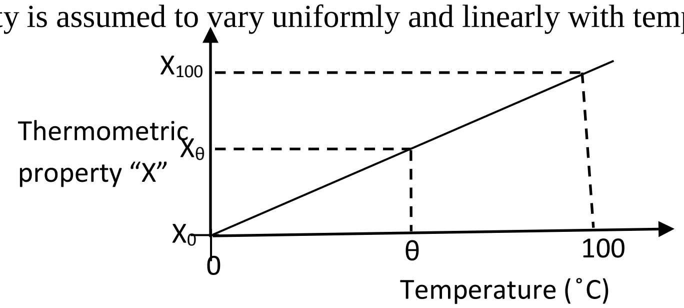
If the property is assumed to vary uniformly and linearly with temperature changes, on Celsius scale of temperature $\theta=\left(\frac{X_\theta-X_0}{X_{100}-X_0}\right)x100^{\circ}C$and on thermodynamic scale of temperature $\theta=\left(\frac{X_\theta}{X_{tr}}\right)x273K$.

Celsius scale of temperature

On this scale of temperature, temperature is measured in degrees centigrade (℃) and makes use of lower fixed point and upper fixed point. Temperatures values on this scale can either be positive or negative.

Establishing a Celsius scale of temperature.

We select a thermometric property and its values X_0 and X_{100} at fixed points i.e. (lower fixed point and upper fixed point) are measured using a specified thermometer and recorded.
The thermometric property $X_\theta$at un known temperature θ is also measured and recorded. If the property is assumed to vary uniformly and linearly with temperature as shown below,



By comparison of slopes,$\frac{X_\theta - X_O -}{\theta} = \frac{X_{100} - X_0}{100}$ we get θ = $\left(\frac{X_\theta - X_0}{X_{100} - X_0}\right) x100°C.$

Thermal dynamic scale of temperature

On this scale of temperature, temperature is measured in "**Kelvin" (K)** and makes use of triple point only. Temperatures values on this scale can also be either positive or negative.

A ***Kelvin*** is $\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of water.

Establishing thermal dynamic scale of temperature

We select a thermometric property and its value X_{tr}at triple point is measured using a specified thermometer and recorded. The thermometric property X_T at un known temperature"**T"** is also measured and recorded.

If the property is assumed to vary uniformly and linearly with temperature changes, on thermodynamic scale of temperature T =$\left(\frac{X_T}{X_{tr}}\right) x273.16K.$

NB: The major advantage of thermal dynamic scale of temperature over Celsius scale is that thermal dynamic scale is suitable for estimating extremely higher temperatures at which gases cease to exist as elements e.g. Temperature of atomic explosions and Temperature of the interior of the star and the sun.

Comparison of scales of temperature

Celsius scale of temperature	**Thermal dynamic scale of temperature**
temperature is measured in degrees centigrade **(℃)**	Temperature is measured in "Kelvin" **(K)**
It makes use of lower fixed point and upper fixed point	It makes use of triple point only

Un known temperature on this scale is given by θ = $\left(\frac{X_\theta - X_0}{X_{100} - X_0}\right) x100°C$.	Un known temperature on this scale is given by T =$\left(\frac{X_T}{X_{tr}}\right) x273K$

Absolute zero temperature

This is the temperature of an ideal gas which corresponds to its zero volume or zero pressure it exerts on the walls of the container in which it is trapped. This value approximates to the triple point of pure water i.e. -273℃ or 0K.

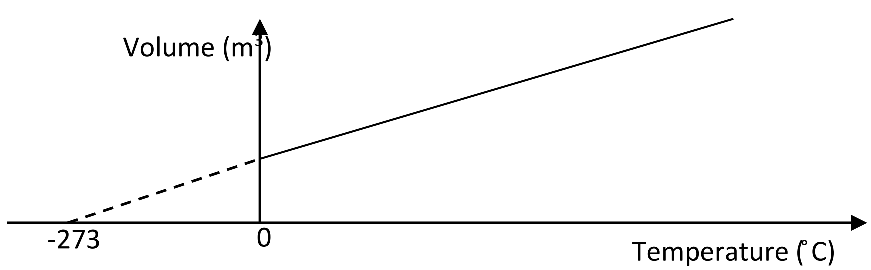
Molecular explanation for existence of absolute zero temperature

When a gas is cooled, its molecules loose kinetic energy continuously since it depends directly on temperature. As molecules loose kinetic energy they move closer into close proximity until when they cease to have kinetic energy. At this point the gas is said to occupy a negligible volume and its temperature at this point is called the absolute zero temperature and the pressure the gas exerts on the walls of the container occupied is negligible.

Estimating absolute zero temperature

A given volume of a fixed mass of an ideal gas is trapped and cooled. During cooling, the gas molecule slow down since their average kinetic energy entirely depends on their thermal dynamic temperatures.

Molecules move closer until when their mean kinetic energy is zero i.e. molecules cannot move any more. When a plot of gas volume "V" is plotted against its thermal dynamic temperature, the following plot is obtained i.e.



Absolute zero temperature is determined by extrapolating the graph until when it touches the temperature axis and is found to be **-273℃ or 0K.**

Thermometers

A thermometer is device for measuring temperature based on its thermometric property. There are several thermometers each with a unique thermometric property as listed below.

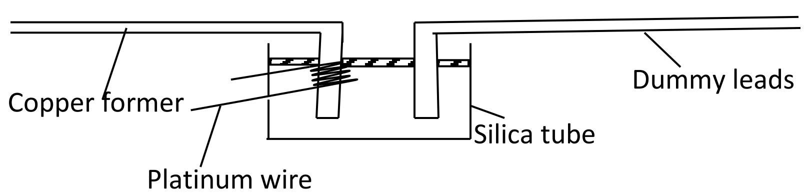
Thermometer	**Thermometric property**
Mercury –in –glass	Length of liquid column
Platinum wire	Electrical resistance
Thermo couple	Electromotive force
Pyrometer	Quality of electromagnetic radiations emitted by a hot body
Constant-volume gas	Pressure of a fixed mass of a gas
Constant- pressure gas	Volume of a fixed mass of gas

Platinum wire thermometer

This uses the principle that the resistance of a metal changes with temperature.

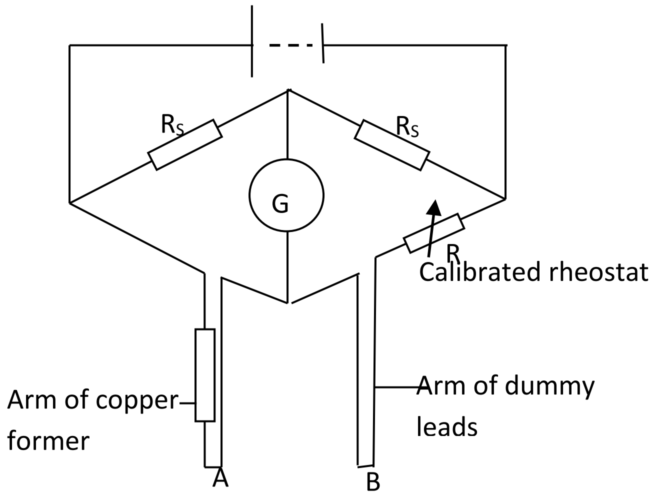
- Structure

It is a thin platinum wire wound on a mica/copper former. Such arrangement is fixed into a silica tube together with dummy leads which are identical to the copper former as shown below.



- Action

The thermometer is connected to Wheatstone bridge as shown below



On Celsius scale

The arm of the platinum wire is immersed in steam and rheostat **R** is adjusted until when the galvanometer reads zero amperes. The value of the rheostat at this temperature $\mathbf{R_{100}}$ is read and recorded.

The above procedure is repeated when the arm of platinum wire is immersed in ice – water mixture and a liquid of unknown temperature **θ** such that the value of the rheostat in each case $\mathbf{R_0}$and $\mathbf{R_\theta}$ are also read and recorded.

Assuming resistance of platinum wire to vary linearly, uniformly and continuously with temperature, the un known temperature θ of the liquid is given by θ =$\left(\frac{R_\theta - R_0}{R_{100} - R_0}\right) x100^\circ C.$

On thermal dynamic scale

The arm of the platinum wire is immersed in a mixture of pure ice, pure water and pure steam existing in equilibrium. The rheostat **R** is adjusted until when the galvanometer reads zero amperes. The value of the rheostat at this temperature $\mathbf{R_{tr}}$ is read and recorded.

The above procedure is repeated when the arm of platinum wire is immersed in a liquid of unknown temperature **T** such that the value of the rheostat $\mathbf{R_T}$is also read and recorded.

Assuming resistance of platinum wire to vary linearly, uniformly and continuously with temperature, the un known temperature θ of the liquid is given by T =$\left(\frac{R_T}{R_{tr}}\right) x273.16K.$

Advantages of platinum wire thermometer

- It is accurate since its resistance varies linearly with temperature.
- It can be used to measure over a wide range i.e. (-200℃ to 1200℃).

Disadvantages of platinum wire thermometer

- Platinum wire has a low conductivity and a high **s.h.c** thus takes long for an observation to be made.

- Due to the size the tube, this thermometer can not be used to measure at a point.
- The thermometer is bulky.

Explain why platinum is preferred to other metals for use in resistance thermometer.

(i) Platinum has a high coefficient of resistance ie a small change in temperature produces appreciable change in resistance.

(ii) Platinum has a very high melting point.

(iii) Platinum is readily available in a state of high purity.

Example

1. The resistance R of platinum wire at temperature θ°C as measured by mercury-in-glass thermometer is given by; $R_\theta=R_0(1+a\theta+b\theta^2)$ where a $=3.8\times10^{-3}K^{-1}$ and b$=-5.6\times10^{-7}K^{-2}$. Calculate the temperature of platinum thermometer corresponding to 200°C on glass scale.

Solutions:

Since both scales agree at fixed points, and that for platinum thermometer $R_\theta = R_o\left(1+a\theta+b\theta^2\right)$ then;

$$\Rightarrow R_{100} = R_o\left(1+a\times100+b\times10000\right)$$

$$R_{200} = R_o\left(1+a\times200+b\times40000\right)$$

$$\therefore \theta = \left(\frac{R_\theta - R_o}{R_{100}-R_o}\right)*100$$

$$\theta = \left(\frac{200a+40000b}{100a+10000b}\right)*100$$

θ=197℃.

Substituting for the values of **a** and**b** given above we get that when the temperature on mercury-in-glass thermometer is 200°C the platinum thermometer indicates 197°C.

2. The resistance of platinum thermometer is 2.04**Ω** at ice point and 3.02**Ω** at the steam point.

i) What should be the temperature of platinum wire so as to have a resistance of 9.24**Ω**?

ii) If a constant-pressure thermometer had been used, the same temperature would correspond to 1040°C. Explain the deviation.

Solutions:

i) $\theta = \left(\frac{9.24-2.04}{3.02-2.04}\right)x100^\circ C$ = 734.7℃.

ii) The variation of platinum resistance with temperature is different to the variation of volume of a fixed mass of dry gas with temperature thus the two thermometers give unique readings.

3.

Thermocouples

Whenever two different metals are in contact, an emf is set up at the point of contact. The magnitude of this emf depends on the temperature at the junction of the two metals and therefore the effect is called seebeck or thermoelectric effect.

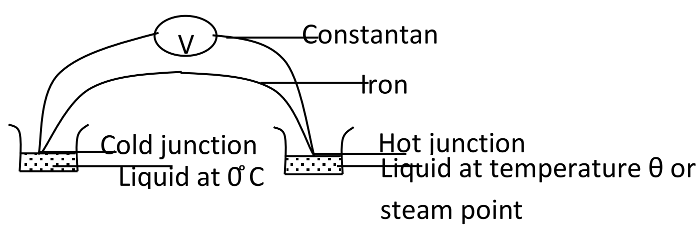
- Structure

Two metal wires of different materials are joined to form a junction.

- Principle

It works on the principle that when two different metals e.g. iron and constantan are joined together with their junctions kept at different temperatures, an e.m.f is induced in the circuit and that this e.m.f depends on the temperature difference between the junctions i.e. $\mathbf{E_\theta = A\theta + B\theta^2 + C\theta^3 + \ldots}$ Where **A, B** and **C** are constants.

- Action



On Celsius scale

The voltmeter reading $\mathbf{E_0}$ is read and recorded when both the cold junction and hot junction are immersed in pure ice. The hot junction is then immersed in pure steam with the cold junction still in ice. The voltmeter reading $\mathbf{E_{100}}$ is read and recorded

With the cold junction still in ice, the hot junction is dipped in a liquid whose temperature is required such that the voltmeter reading $\mathbf{E_\theta}$ is read and recorded. Assuming e.m.f generated to vary linearly, uniformly and continuously with temperature, $\theta = \left(\frac{E_\theta}{E_{100}}\right) x 100^\circ C$ for $E_0 = 0V$.

On thermal dynamic scale

The hot junction of the thermal couple is immersed in a mixture of pure ice, pure water and pure steam existing in equilibrium. The voltmeter reading $\mathbf{E_{tr}}$is read and recorded.

The above procedure is repeated when the hot junction of the thermal couple is immersed in a liquid of unknown temperature **T** such that the voltmeter reading $\mathbf{E_T}$is also read and recorded. Assuming e.m.f generated to vary linearly with temperature, $T = \left(\frac{E_T}{E_{tr}}\right) x 273.16K$.

Conditions for e.m.f to be generated between junctions

- The junctions must be of different metals
- The junctions must be at different temperatures

Sources of errors when using a thermo couple

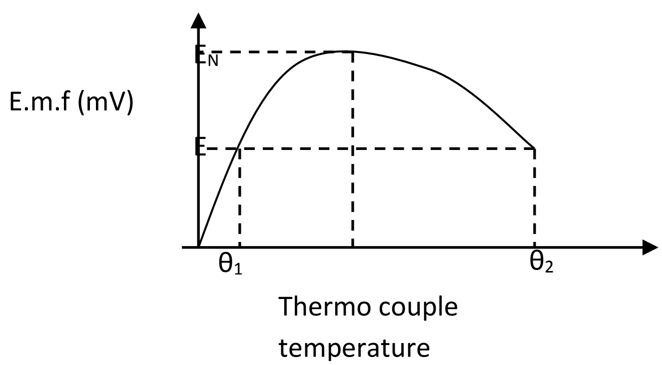
- Leakages due to poor insulation means
- Leakages due to strained thermocouple wires
- Stray thermo electric e.m.fs due to several contacts involved during measurements

Advantages of thermocouples

- The junction can be made pointed such that it can be used to measure temperature at a point.
- Metals used always have a high conductivity and low s.h.c thus suitable for measuring rapidly varying temperatures.
- The thermometers are portable since they are robust and compact.
- It can be made to measure temperature directly by connecting it to an ammeter calibrated to read temperature.

Disadvantages of thermocouples

- E.m.f of a thermocouple does not vary linearly with temperature.
- Thermocouples can give two similar reading readings corresponding to different temperature values as shown below.



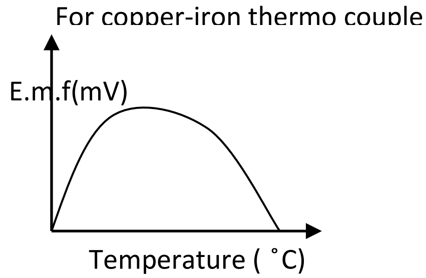
NB:

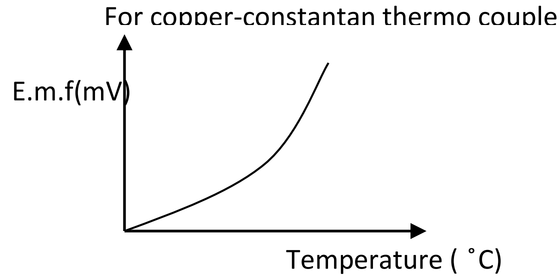
Temperature corresponding to"E_N" is called the **neutral point** of thermocouple.

Neutral temperature of thermocouple is themaximum temperature of thermo couple just below which its e.m.fs vary fairly linearly.

For this very reason, thermo couples are only used as thermometers at temperatures below its neutral temperature.

Variation of e.m.fs with temperature





Explain why a thermocouple can be used to measure rapidly fluctuating temperatures

This is because it has a very small heat capacity and therefore, it has very little effect on the temperature of the body being measured.

Liquid –in –glass thermometer e.g. clinical thermometer

Liquids which are commonly used in such thermometers are; mercury and alcohol to a small extent. The choice of a liquid to be used is based on the following properties;

For mercury

- It is opaque thus readily seen making it readable
- It is a good conductor of heat thus responds quickly to temperature changes
- It expands uniformly making readings to be steady.
- It has a high boiling point implying that it can be used to measure high temperature.

For alcohol

- It is highly expansive making it suitable for measuring high temperatures

Why to a large extent alcohol is not used?

- Alcohol is colorless making it difficult to be read.
- Alcohol is a poor conductor of heat leading to slow responses by the thermometers
- Alcohol expands non uniformly leading to fractuating responses by the thermometer.
- Alcohol has a low boiling point making it un suitable for measuring high temperature.

Why water is not used as thermometric liquid?

- Water is colorless making it difficult to be read.
- Water expands non uniformly leading to fractuating responses by the thermometer.
- Water has a high boiling point and high s.h.c making it un suitable for measuring small temperatures.
- Water has a small range of expansion thus used to measure temperature in a small range e.g. 0℃ and 100℃.

Operation

- **On Celsius scale**

The thermometer is dipped in pure ice and the length of mercury column l_0 is read and recorded. The thermometer is dipped in pure steam and the length of mercury column l_{100} is read and recorded. The thermometer is dipped in a liquid whose temperature θ is required such that the length of mercury column l_θ is also read and recorded. For uniform variation of mercury column with temperature, the un known temperature θ =$\left(\frac{l_\theta - l_0}{l_{100} - l_0}\right) x100^\circ C.$

- **On thermal dynamic scale**

The thermometer is immersed in a mixture of pure ice, pure water and pure steam existing in equilibrium. The length of mercury column $\mathbf{l_{tr}}$is read and recorded.
The above procedure is repeated when the thermometer is immersed in a liquid of unknown temperature **T** such that the length of mercury column $\mathbf{l_T}$is also read and recorded. Assuming the length of mercury column to vary linearly with temperature, T =$\left(\frac{l_T}{l_{tr}}\right) x273.16K.$

Advantages of liquid-in-glass thermometer.

- The thermometer is simple, cheap
- and portable.
- The thermometer can be calibrated to give direct readings e.g. clinical thermometer

Question:

1. The fundamental interval of glass-in- glass thermometer is 192mm.Calculate the temperature on Celsius scale corresponding with mercury thread of 67.2mm long.

Solutions:

l_0 =0.0mm, l_{100}= 192.0mm and l_θ =67.2mm

For θ =$\left(\frac{l_\theta - l_0}{l_{100} - l_0}\right) x100^\circ C$, θ =$\left(\frac{67.2-0.0}{192.0-0.0}\right) x100^\circ C.$

θ =35℃

2. One junction of a thermocouple is placed in melting ice while the other is inserted into a bath whose temperature as measured by a high temperature mercury-in-glass thermometer at 760mmHg is as shown in the table below.

T/℃	0.0	100.0	200.0	300.0	400.0	500.0
e.m.f/mV	0.0	0.64	1.44	2.32	3.25	4.32

By Graphical method determine;

i) Temperature as read from the thermocouple thermometer corresponding to 308^{o}C on mercury-in-glass thermometer
ii) Temperature as read from mercury-in-glass thermometer corresponding to 250^{o}C on the thermocouple thermometer.

Solutions:

i) From the graph, E_{380}=3mV mercury-in-glass thermometer.

$\Rightarrow\theta$ on the thermocouple scale giving 3mV

$$\theta = \left(\frac{3.0-0.0}{0.64-0.0}\right) x100^{\circ}C = 458.9^{\circ}C.$$

ii) On the thermocouple scale

$$250.0 = \left(\frac{E_{\theta}-0.0}{0.64-0.0}\right) x100^{\circ}C,$$ from which E_{θ}=1.6mV.

$\Rightarrow$On mercury-in-glass thermometer

From the graph the value of θ for which $E_{\theta} = 1.6mV$ is 220°C

3.

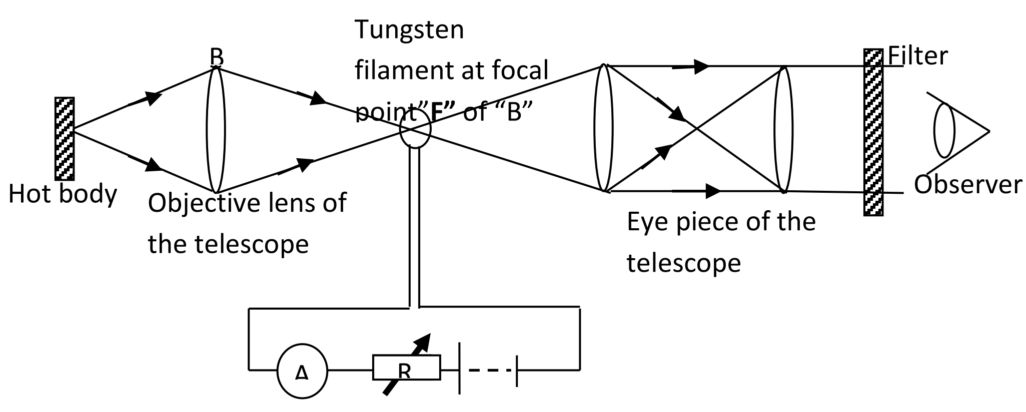
Pyrometry

This is the measurement of very high temperatures.

Radiation Pyrometers

A radiation pyrometer is an instrument used to measure high temperatures of a body based on the radiations emitted by the body. Such instruments are of two types i.e. optical pyrometer which responds to only visible radiation and total radiation pyrometer which responds to both visible and infrared radiation.

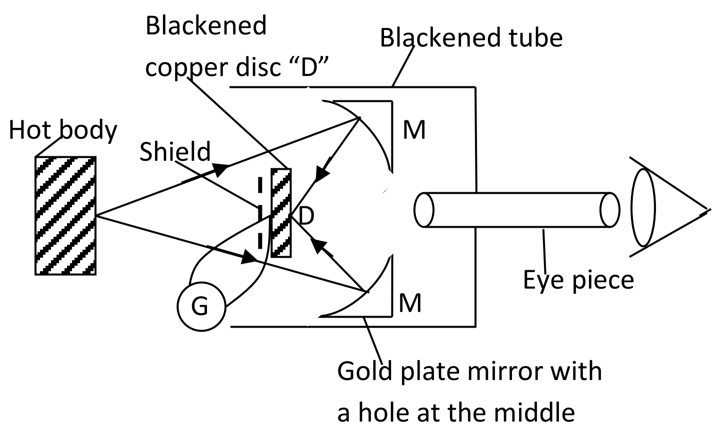
Optical pyrometer



The eye piece is focused on the filament receiving radiations from a body whose temperature is to be determined. The objective lens is adjusted until when image of the body lies in the same

plane with the filament. At this point both the object and the filament appear red hot as seen through the filament placed at the focal point of the objective lens of the telescope.
Current through the filament is adjusted by varying R until when the filament and the image of the body have the same appearance i.e. when the object just disappears from the back ground of the filament. Temperature of the hot body is read from the ammeter **"A"** calibrated to measure temperature in **"Kelvin".**

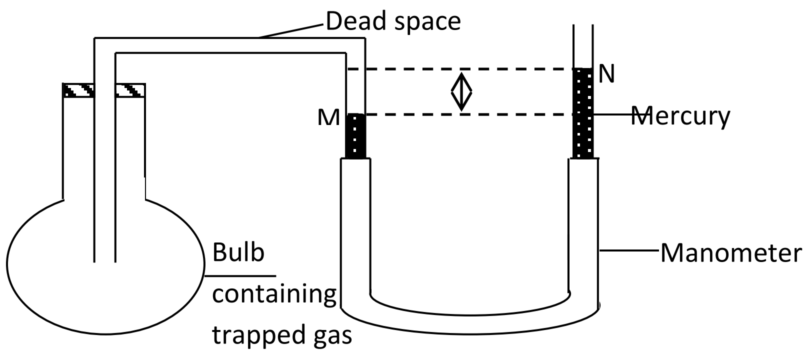
Total radiation pyrometer



Radiations from the hot body fall on mirror "M" and get reflected to the copper disc "D". The disk is heated by the radiations until it reaches an equilibrium temperature i.e. temperature at which its rate of heat loss = its rate of heat gain.
The deflection "θ" of the galvanometer is read and recorded and is proportional to the energy "**E"** radiated by the black body given by Stefan's law of black bodies i.e. $E\alpha T^4$ or $\theta\alpha T^4$…….(i)
The pyrometer is now focused on molten gold at its melting point **($T_{AU}\approx 1063$ ℃).** The deflection θ_{AU} is read and recorded and is proportional to T_{AU}^4 i.e. $\theta_{AU}\alpha T_{AU}^4$……(ii).
Temperature "T" is calculated using the expression $T = \sqrt[4]{\frac{\theta}{\theta_{AU}} T_{AU}^4}$.

Gas thermometers

Constant –volume- gas thermometer



- **On Celsius scale**

The bulb is immersed in pure ice and after some time the manometer is adjusted to bring mercury to mark "M" (to ensure constant volume of the trapped gas). The difference in the manometer readings "h_0" is measured using a metre rule and recoded.
The above procedure is repeated when the bulb is immersed in pure steam and a liquid of unknown temperature **θ** such that the differences in the manometer readings "h_{100}" and "h_θ"in each case are measured using a metre rule and recoded.
Pressure the gas exerts on mercury in the closed limb of the manometer in each case is given by
P =H-h for "N" below "M" or P =H+h for "N" above "M".
For uniform variation of mercury column with temperature, the un known temperature

$$\theta = \left(\frac{h_\theta - h_0}{h_{100} - h_0}\right) x 100^\circ C.$$

- **On thermal dynamic scale**

The bulb is immersed in a mixture of pure ice, pure water and pure steam existing in equilibrium. The difference in the manometer readings **"h_{tr}"** is measured using a metre rule and recoded.
The above procedure is repeated when the bulb is immersed in a liquid of unknown temperature **T**. The difference in the manometer readings **"h_T"** is measured using a metre rule and recoded.
Assuming the length of mercury column to vary linearly with temperature, T $=\left(\frac{h_T}{h_{tr}}\right) x 273.16K.$

Sources of errors

(i) Thermal expansion of the bulb.
(ii) Capillary effects at the mercury surfaces.
(iii)Air is not an ideal gas.

(iv) The gas in the ddead space is not at the temperature as that being measured.

Such corrections are carried out as follows;

- A predetermined estimated correction of temperature due to expansion of the bulb is added to the observed temperature measured by the thermometer.
- The dead space is made too small so as to contain a small fraction of the total mass of the gas to minimize temperature changes due to much air in the dead space.
- Temperature at different points in the dead space is measured using mercury thermometer such that effects of air in such space are catered for in the calculations.
- Temperature in the manometer and barometer is measured using mercury thermometer such that it is catered for in the calculations

Example

1. In constant volume gas thermometer, the following observations were recorded on a day when the barometric reading was 760mmHg.

	Reading in closed limb (mm)	Reading in open limb (mm)
Bulb in pure ice	126	112
Bulb in steam	126	390
Bulb at room temperature	126	157

i) State the thermometric property of the thermometer

ii) Calculate temperature as measured by the thermometer

Solutions:

b) ii) From the table h_o=112-126=-14mmHg

h_θ=157-126=31mmHg

h_{100}=390-126=264mmHg

$$\theta = \left(\frac{h_\theta - h_o}{h_{100} - h_o}\right) \times 100 = \left(\frac{31+14}{264+14}\right) \times 100 = 16.18^o C$$

2. The volume of some air in a constant-pressure thermometer and length of an Iron rod are measured at 0^oC and at 100^oC and the following results where obtained.

	0^oC	100^oC
Volume of air	28.5	38.9
Length of rod	100.00	100.20

Calculate (i) absolute zero temperature of this air scale

(ii) The length of the Iron rod at this temperature given that $l_\theta=l_0(1+\theta\alpha)$.

Solutions:

i) At absolute zero, $V_\theta=0.0m^3$

$$\theta=\left(\frac{0-28.5}{38.9-28.5}\right)\times 100=-274^oC$$

ii) Using $l_\theta=l_o(1+\alpha\theta)$

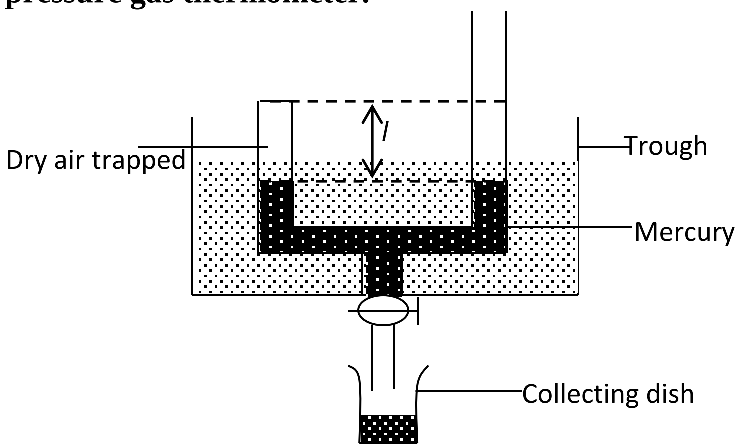
$$100.20=100(1+\alpha*100)$$

$\alpha = 2.0x10^{-5}$ ℃$^{-1}$

$$\therefore l_{-274}=100(1-2.0*10^{-5}*274)$$

$=99.450m$

Constant –pressure gas thermometer.



- **On Celsius scale**

Dry air is trapped in the closed limb of the tube of uniform cross-section area. The tube is immersed in pure ice and by the help of the tap, mercury in both limbs is leveled by letting some out (to ensure constant pressure/same pressure at both ends) and the length of air column l_0 is measured using a meter rule and recorded.

The tube is now immersed in (a) pure steam and (b) liquid whose temperature θ is required such that the lengths of air column in each case l_θ and l_{100} are measured and using a meter rule and recorded. Assuming the variation of air column to vary linearly with temperature, un known temperature

$$\theta=\left(\frac{l_\theta-l_0}{l_{100}-l_0}\right)x100^\circ C.$$

- **On thermal dynamic scale**

The tube is immersed in a mixture of pure ice, pure water and pure steam existing in equilibrium. The length of the air column ***"l_{tr}"*** is measured using a metre rule and recoded.

The above procedure is repeated when the tube is immersed in a liquid of unknown temperature **T**. The length of the air column ***"l_T"*** is measured using a metre rule and recoded.

Assuming the length of air column to vary linearly with temperature, T =$\left(\frac{l_T}{l_{tr}}\right) x273.16K$.

Sources of errors in gas thermometers

- Air is not ideal yet the thermometer assumes ideal gases.
- Air in the tube may not be at the same temperature throughout it and this makes it difficult to level up mercury.

Advantages of gas thermometers

- Gas thermometers are very accurate since their thermometric properties vary linearly with temperature.
- Gas thermometers measure temperature over a wide range.

Disadvantages of gas thermometers

- Gas thermometers are very bulky
- Gas thermometers have a slow response to temperature changes because gases are poor conductors of heat.
- Gas thermometers can not give direct readings

NB:

- The fact that thermometric properties of gas thermometers vary linearly with temperature, gas thermometers are very accurate thus used as standard thermometers for calibrating other thermometers.
- Constant –volume gas thermometers are preferred to constant –pressure gas thermometers because it is easier to measure accurately changes in pressure than changes in volume

Assignment

1) Temperature **θ** of a liquid is determined using a resistance thermometer and a constant-pressure gas thermometer and the following measurements were obtained.

R_o= 2.00Ω, R_{100}=2.50 Ω, R_θ=2.09 Ω

V_{0} = 4.00m^3, V_{100}=5.50 m^3, V_θ=4.25 m^3

Determine the value of θ for each thermometer and account for their discrepancy

2) The resistance R of platinum wire at various temperatures on a mercury-in-glass scale is given by

$T/^oC$	0	20	40	60	80	100
R/Ω	10.0	10.26	10.6	11.02	11.48	12.00

Use graphical method to determine;

(i) Temperature as read from platinum wire thermometer corresponding to 65^oC on mercury-in-glass thermometer

(ii) Temperature as read from mercury-in-glass thermometer corresponding to 45^oC on platinum wire thermometer.

3) A particular resistance thermometer has a resistance of 30Ω at ice point, 41.58Ω at steam point and 34.59Ω when immersed in a boiling liquid and constant-volume-gas thermometer reads 1.33×10^5pa, 1.62×10^5pa, 1.528×10^5pa at the three points stated above respectively. Calculate temperature of the boiling liquid as measured by each thermometer.

4 (a)(i) Define the terms heat and temperature.

(ii)State four quantities of a good thermometric property.

(b)Describe how you would estimate absolute zero temperature.

(c) Describe how unknown temperature of a liquid can be measured by liquid-in-glass thermometer.

(d) Use kinetic theory of gases to explain the existence of absolute zero temperature.

(e) A copper-constantan thermocouple with its cold junction at 0℃ had an e.m.f of 4.28mV when the other end is immersed in pure steam and 9.29mV when the second end is immersed in a liquid at 200℃. Given that its e.m.f is related to temperature difference θ by the expression $\mathbf{E_\theta = A\theta + B\theta^2}$ determine the values of A and B.

5(a)(i) Define the terms Fixed point and Lower fixed point.

(ii) In tabular form state any four kinds of thermometers with their corresponding thermometric property.

(b)(i) Describe the structure of platinum resistance thermometer.

(ii)Explain why constant –pressure thermometer gives a slow response.

(c) One Junction of a thermocouple is placed in melting ice while the other is inserted into a bath whose temperature as measured by high temperature mercury- in- glass thermometer is T°C as shown in the table below.

T//°C	0	101	212	302	425	552	635

e.m.f/mV	0	0.75	1.40	2.33	3.22	4.35	5.14

By graphical method find;

(i) Temperature as measured from thermocouple thermometer corresponding to 380°C on mercury-in-glass thermometer.

(ii) Temperature as read from mercury-in-glass thermometer corresponding to 250°C on the thermocouple thermometer.

6(a)(i) Define a thermometer.

(ii) Describe how unknown temperature of a liquid can be determined by platinum resistance on thermodynamic scale.

(b)(i) State two advantages and two disadvantages of thermocouples.

(ii) In constant-volume gas thermometer, the following observations were recorded on a day when barometric reading was 760mmHg.

	Reading in closed limb (mm)	**Reading in the open limb (mm)**
Bulb in pure ice	130	123
Bulb in steam	130	165
Bulb at room temp	130	178

State the thermometric property of the thermometer and calculate the temperature value as measured by the thermometer.

(c) Explain the following in relation to liquid – in –glass thermometer.

(i) Mercury is the best liquid for measuring high temperatures.

(ii) Alcohol is the best liquid for measuring small temperatures.

(d) Sketch graphs of e.m.fs against temperature for;

(i) Copper – iron thermocouple.

(ii) Iron-constantan thermocouple.

7(a)(i) Define neutral point of a thermocouple.

(ii) Describe how total radiation pyrometer can be used to determine temperature of a very hot body.

(b)(i) State the principle of operation of the thermocouple.

(ii) Temperature of a liquid is determined using a resistance thermometer and constant – pressure gas thermometer with the following measurements.

$R_o = 4.53\Omega$, $R_{100} = 8.54\Omega$ $R_\theta = 7.59\Omega$

$V_o = 4.02m^3$ $V_{100} = 5.85cm^3 V_\theta = 4.52m^3$

Calculate the temperature value as measured by each thermometer and explain the discrepancy

8 (a)(i) State the requirements for establishing thermal dynamic scale of temperature.

(ii) Explain why scales of temperature based on different thermometric properties may not agree.

b) (i) With the aid of a labeled diagram, describe how a constant volume gas thermometer is used to measure temperature on thermal dynamic scale.

(ii) Give disadvantages and advantages of gas thermometers.

c) If a wire has a resistance of 4Ω at the triple point of water, find its resistance at 80^oC.

d) (i) What are Pyrometers?

(ii) With the aid of a diagram, describe how the optical pyrometer is used to measure temperature of a hot body.

9 (a) What is meant by the following terms;

i) Absolute zero temperature

ii) Triple point of water

b) i) Describe the steps taken to establish scales of temperature.

ii) State three advantages of thermocouple over an electrical resistance thermometer

c) With a well labeled diagram describe how room temperature can be measured using platinum wire thermometer.

d) i) State and explain the source of inaccuracies while using mercury-in glass thermometer

10(**a**) Explain the following observation as applied to gas thermometer

i) Constant-volume gas thermometer is preferred to constant –pressure gas thermometer.

ii) Gas thermometers are used to calibrate other thermometers.

b) (i) Explain why it is possible for two different bodies at different temperatures may have the amount of heat.

(ii) Resistance of platinum wire is $\mathbf{6.750R_o\Omega}$ at room temperature. Calculate the value of room temperature if temperature coefficient of platinum is $2.07x10^{-4}K^{-1}$.

c) Resistance of a metal wire at temperature θ measured on standard scale is given by $\mathbf{R_\theta=R_0(1+A\theta+B\theta^2)}$ where **A** and**B** are constants. Given that $\mathbf{B = 10^{-3}A}$, calculate the temperature value on the resistance thermometer corresponding to 60^0C on standard scale.

d) (i) State corrections made on constant-volume gas thermometer before being used to measure temperature.

(ii) Explain how the corrections mentioned in d(i) above are carried out.

Calorimetry

Heat absorbed by a body of mass **m** at temperature **θ** is given by

$Q = mc\Delta\theta = C\Delta\theta$ where **c** is the specific heat capacity and **C** is heat capacity. This implies that;

Heat capacity "**C**" is the amount of heat required to raise temperature of any given mass of a substance by 1K.

$$C = \frac{Q}{\Delta\theta}$$

Specific heat capacity "**c**" is the amount of heat required to raise the temperature of 1kg mass of a substance by 1K.

The specific heat capacity of water is $4200Jkg^{-1}K^{-1}$, this means that 4200J of energy is required to raise the temperature of 1kg mass of a substance by 1 kelvin.

NB: Hydrogen is used as a cooling gas for enclosed electric generators because it has a high **s.h.c** and is highly conductive.

- Liquids have a higher **s.h.c** than solids because liquids are less dense with lower conductivity than solids.
- Wet foods stay hot longer than dry foods because; water in wet food has a high **s.h.c** which enables it to maintain heat in wet foods for a longer time than in dry foods.
- The sea remains colder than land during day and hotter than land during night because;

⇒The high **s.h.c** of water than that of land enables it to absorb less heat than that absorbed by land during day. This much heat absorbed by land heats up air above its surface making to rise as it gets replaced by the cooler air from the sea thus **sea breeze**

.

⇒At night both land and sea are in a cooling state but since the **s.h.c** of land is lower than that for water, land cools faster than sea. This makes air above the sea surface to be heated up by the released heat absorbed during the day making it to rise as it gets replaced by the cooler air from land thus **land breeze**.

Heat losses in calorimetry can be reduced by:

(i) Polishing the calorimetr to reduce heat loss by radiation.
(ii) Supporting it on an insulating stand to minimize conduction.
(iii) Surrounding the calorimeter by a jacket of a poor heat conductor to reduce convection and conduction.

Example

1. Water flowing at a speed of $5ms^{-1}$ falls over a 50m high water fall into a still pool below. Calculate the approximate rise in temperature of the water.
   Solution
   Let m be the mass of water, loss of mechanical energy = mgh+ $\frac{1}{2}mv^2$, thus gain of heat energy = $mC\Delta\theta = \text{ mgh } + \frac{1}{2}mv^2$, thus
2. A silver bullet with a speed of $500ms^{-1}$ initially at a temperature of 20^0C stops suddenly and all its energy is converted into thermal energy. What is its temperature rise? (s.h c = $243Jkg^{-1}K^{-1}$)
   Solution
   Mechanical energy of the bullet = $\frac{1}{2}mv^2 = increase\ in\ internal\ energy, thus$
   $$\frac{1}{2}mv^2 = mC\Delta\theta, thus\ \ \Delta\theta = \frac{v^2}{2C} =$$

Methods for determining specific heat capacity

a) For solids we use;

- Method of mixtures
- Electrical method

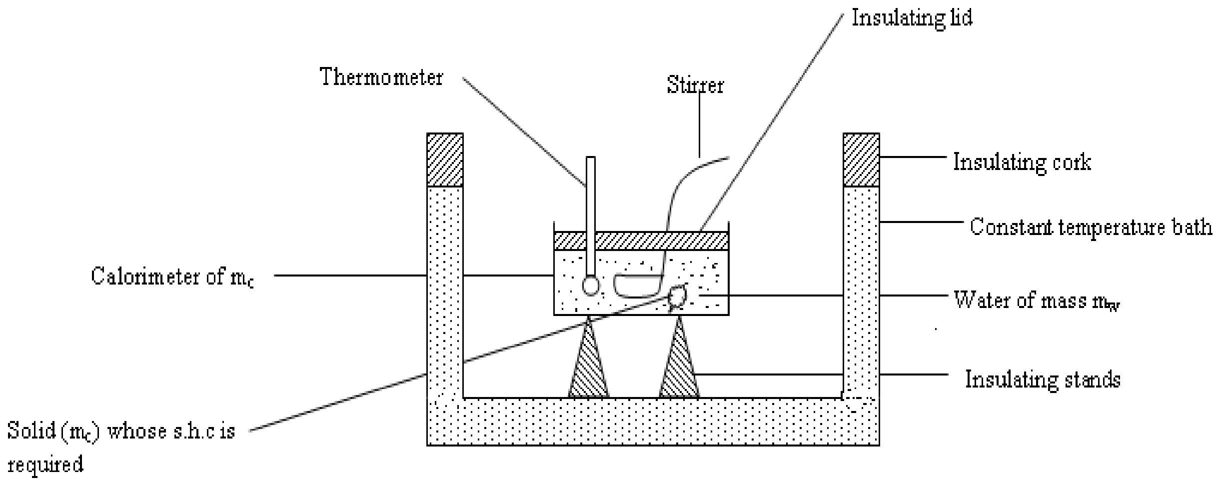
b) For non volatile liquids we use;

- Method of mixtures
- Electrical method
- Continuous flow method.

For solids

Method of mixtures

A known mass $\mathbf{m_s}$ of solid is heated to a known temperature $\boldsymbol{\theta_1}$ and quickly transferred into water of known mass $\mathbf{m_w}$ and known specific heat capacity $\mathbf{c_w}$contained in a calorimeter of known mass $\mathbf{m_c}$ and known specific heat capacity $\mathbf{c_c}$ at a common temperature $\boldsymbol{\theta_2}$. The mixture is stirred gently until a new common steady temperature of mixture $\boldsymbol{\theta_3}$ is obtained. This temperature is read from the thermometer and recorded.



Assuming no heat loss to the surrounding then;
Heat lost by the solid = Heat absorbed by (water + calorimeter) i.e.
$m_s c_s(\theta_1 - \theta_3) = m_w c_w(\theta_3 - \theta_2) + m_c c_c(\theta_3 - \theta_2)$, from which c_s can be obtained.

Precautions

- The specimen must be transferred as fast as possible but with care to avoid splashing of water from calorimeter.
- The calorimeter must be insulated and placed on an insulating stand in a constant temperature bath.
- The calorimeter must be polished on its inner and outer surface to reduce heat loss by radiation.
- Stirring must be done to ensure uniform distribution of heat.

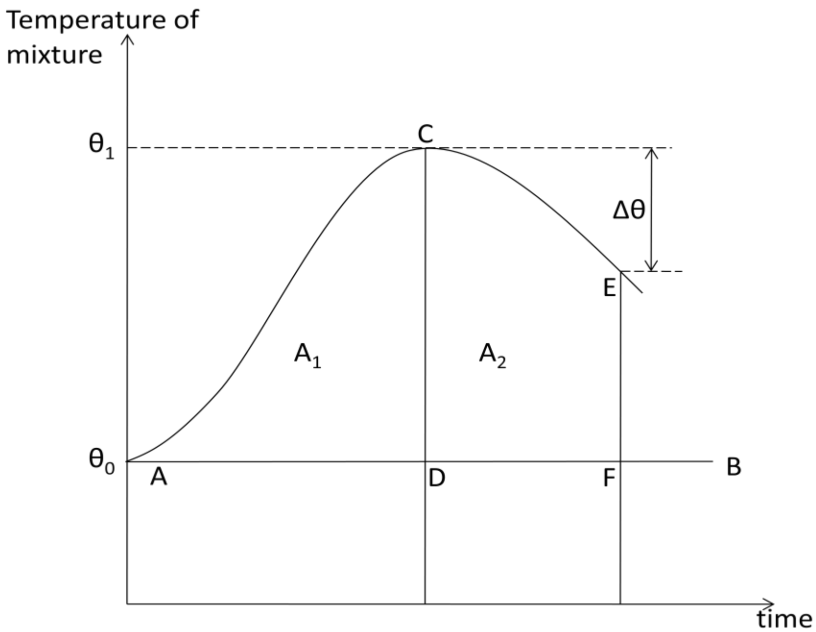
Cooling correction

Cooling correction is the temperature value added to the maximum observed temperature of a mixture whose temperature is being raised so as to cater for heat loses to the surroundings while the temperature of the mixture was rising or in the number of degrees celciuos that should be added to the observed maximum temperature to cater for the heat lost to the surrounding during temperature rise.

Determination of cooling correction

A rubber bung is placed in water being heated and heating is continued until the boiling point of water is reached.
It is then removed and shaked to remove water drops which clings on it and then gently transferred to a calorimeter which contains water at room temperature, θ_0 and stirred.

The temperature of the mixture is recorded for every after ½ minute until the temperature of the mixture has fallen by about 1ºC below the observed maximum temperature, θ_1.
A graph of temperature against time is plotted.



A line AB through θ_0 parallel to the time axis is drawn. A line CD through θ_1 parallel to the temperature axis is drawn. A line EF beyond CD parallel to the temperature axis is drawn and $\Delta\theta$ noted. The areas A_1 and A_2 under the graph are estimated by counting the squares on the graph paper. The cooling correction, θ, is given by , $\theta = \frac{A_1}{A_2} \times \Delta\theta^oC.$

NB: Generally average temperature fall (cooling correction)

e = $\frac{1}{2}$(Rate of temperature fall)x(Time of heating) i.e. e =$\frac{1}{2}$ (R_fxT_h**).**

Factors that determine the rate of loss of heat from a body.

a) The temperature of the surface of the body.
b) The temperature of the surrounding.
c) The area of the surface of the body.
d) The nature of the surface.
e) The pressure of the surrounding.
f) The volume and shape of the enclosure.

Example:
75g of a liquid is placed in copper calorimeter of mass 50g at 17.2℃. A heater of negligible thermal capacity is immersed in a liquid and operated at 1.8A and 6.3V for 4minutes such that

temperature is raised to 25℃. Subsequently temperature falls to 24.7℃ after 2minutes since heating was stopped. Calculate s.h.c of the liquid given that s.h.c of copper is 0.42$Jg^{-1}K^{-1}$.

Solutions

Cooling correction $=\frac{1}{2}\left(\frac{25.0-24.7}{2}\right)x4$ =0.3℃.

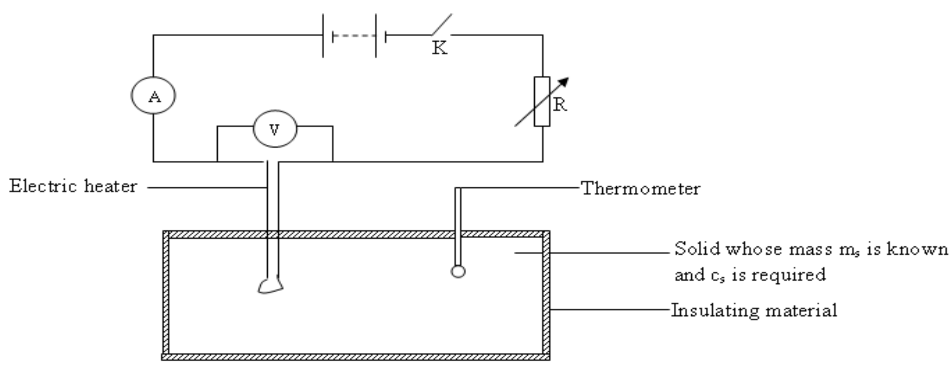
Corrected temperature rise = 25.3 -17.2 =8.1K

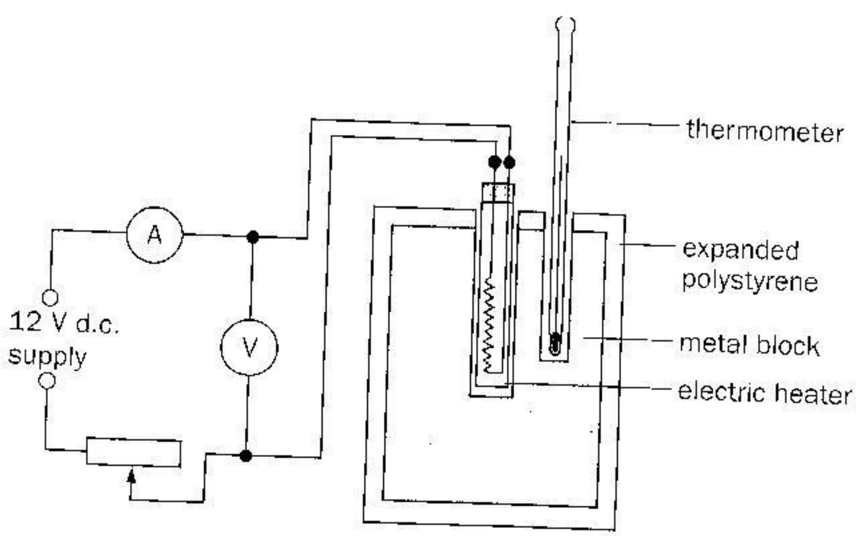
Heat supplied by the heater = Heat absorbed by (liquid + copper)

6.3x1.8x4x60 =50x0.42x8.1 + 75x8.1xc_l

c_l= 4.2$Jg^{-1}K^{-1}$

Electrical method





Two holes are drilled into the solid whose specific heat capacity in required.

The mass of the block is m is determined.

The solid is then lagged and the initial temperature θ_0 is reorded.

The swith k is closed and the solid is heated for a time t.
The values of I and V are read and recorded and the maximum observed temperature **θ_1** is noted.
Assuming no heat is lost to the surrounding, electrical energy produced by the heater = the heat received by the block$\Longrightarrow IVt = mC(\theta_1 - \theta_0) \Longrightarrow C = \frac{IVt}{m(\theta_1-\theta_0)}$
NB:

Example

1) An electrical heater supplies 500J of heat energy to a copper cylinder of mass 32.4g. Find the increase in temperature of the cylinder (s.h.c of Cu=385$Jkg^{-1}K^{-1}$) assuming no heat loss to the surrounding.

Solutions:
Heat supplied by the heater=Heat gained by the cylinder

$500 = 32.4x10^{-3}x385x\Delta\theta$

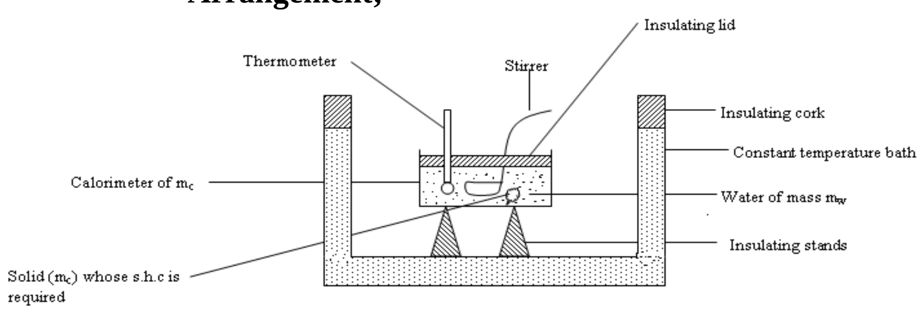
$\Delta\theta = \frac{500}{385x32.4x10^{-3}} = 40.08℃$

2. An electrical heater rated 48W,12V, is placed in a well insulated metal of mass 1.0kg at a temperature of 18^0C. When the power is switched on for 5minutes, the temperature of the metal rises to 34^0C. Find the specific heat capacity of the metal.

For non volatile liquids

Method of mixtures

Arrangement;

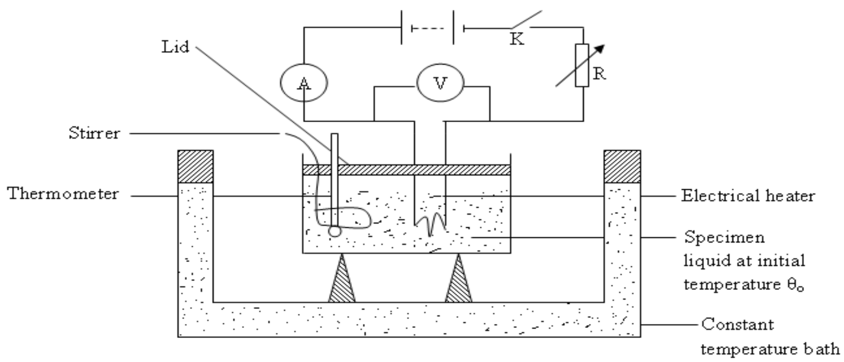


A solid of known heat capacity **C_s** is heated to a known temperature **θ_1** and quickly transferred into a calorimeter of known massheat capacity**C_c**containing a liquid of known mass **m_l** whose specicic heat capacity **c_l** is required at common temperature **θ_2**.
The mixture is stirred gently until when a new steady common temperature **θ_3** of the mixture is obtained. This temperature **θ_3**is read from the thermometerand recorded**.** If heat losses to the surrounding are negligible we get that; Heat lost by the solid = Heat absorbed by (liquid+calorimeter) i.e. **$C_s(\theta_1-\theta_3) = m_lc_l(\theta_3-\theta_2)+C_c(\theta_3-\theta_2)$** from which **$c_l$** is determined.

Disadvantages of the above method

- There is heat loss to the surrounding through convection, radiation and conduction.
- There is likely to be an increase in the mass of water in the calorimeter because some water clings to the surface of the solid

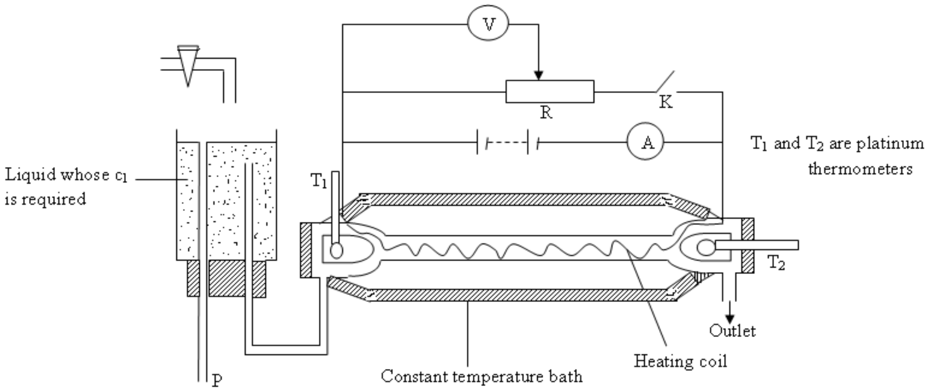
Electrical method



A heating coil is immersed in a calorimeter of known heat capacity, containing a specimen liquid. The temperature $\boldsymbol{\theta}_0$ of the liquid is measured.
A steady current $I_1$is passed through the coli for a measured time,t. the p.d, V_1 across the heating coil is recorded. The temperature $\boldsymbol{\theta}_1$ of the liquid is measured after thorough stiring.

Therefore, $I_1V_1 = m_l c_1(\theta_1-\theta_0)+C_c(\theta_1-\theta_0)+h$……..(i)
K is opened, R is set to a new value and K is reclosed. After the same time of heating as before, the ammeter, voltmeter and thermometer values of I_2, V_2 and θ_2 are read and recorded such that;
$I_2V_2 = m_l c_2(\theta_2-\theta_0)+C_c(\theta_2-\theta_0)+h$……. (ii)
Combining equations (i) and (ii) yields $c_l = \frac{(V_2I_2-V_1I_1)t-C_c(\theta_2-\theta_1)}{m_l(\theta_2-\theta_1)}$.

Continuous flow method



The rate of flow of liquid is adjusted to some value by either lowering or raising P.

The liquid is heated by the coil on closing K and when the two thermometers read steady temperatures $\boldsymbol{\theta_1}$ and $\boldsymbol{\theta_2}$, the ammeter and voltmeter readings $\mathbf{I_1}$ and $\mathbf{V_1}$ and the time of heating **t** are read and recorded. The mass m_1 of the liquid collected is determined by measuring cylinder and recorded such that for uniform rate of fluid flow,

$$I_1V_1t = m_1c_l(\theta_2-\theta_1) + h \;\ldots\ldots\; (i)$$

K is opened, the rate of flow is increased by raising P and Current in the circuit is adjusted by re-setting R such that $(\boldsymbol{\theta_2-\theta_1})$ is kept constant to ensure the same heat loss **h** in the same time interval **t**. The ammeter and voltmeter readings $\mathbf{I_2}$and$\mathbf{V_2}$ are read and recorded. The mass $\mathbf{m_2}$ of the liquid collected is determined using a measuring cylinder and recorded such that for uniform rate of fluid flow,

$$I_2V_2 = m_2c_l(\theta_2-\theta_1) + h\ldots\ldots(ii)$$

Solving (i) and (ii) simultaneously yields $c_l = \frac{(I_1V_1-I_2V_2)t}{(m_1-m_2)(\theta_2-\theta_1)}$ or, $c_l=\frac{(I_1V_1-I_2V_2)}{(m'_1-m'_2)(\theta_2-\theta_1)}$

Where; $m'_1=\frac{m_1}{t}$ and $m'_2=\frac{m_2}{t}$.

Precautions

- Readings must be taken at steady state
- Platinum resistance thermometer should be used
- Experiment is repeated to cater for heat loses to the surrounding
- The potential difference across the coil is measured accurately by the potential meter
- The apparatus is evacuated to prevent heat loss by convection
- The rate of flow of the liquid (water) should be constant

Advantages of continuous flow method

- Specific heat capacities of the apparatus need not to be known since readings are read at steady state.
- The presence of a vacuum around a glass tube minimizes heat losses by convection and conduction.
- The inlet and outlet temperatures are measured at steady states thus increased accuracy.

- No cooling correction is required since heat losses to the surrounding can be eliminated by repeating the experiment.

Disadvantages of continuous flow method

- The apparatus is bulky.
- The method can only work with liquids.
- Large quantities of the liquid are needed since liquid flow is continuous.
- It is really very hard to maintain constant temperature difference ($\boldsymbol{\theta}_2$-$\boldsymbol{\theta}_1$) at both ends.

Advantages of continuos flow method over method of mixtures

a) In the continuos flow method, heat capacity of the apparatus needs not to be known unlike the method of mixtures.
b) Heat losses to the surrounding can be accounted for in continuos flow method unlike in the method of mixtures.
c) Heat loses are greatly minimized in the continuos flow method since the liquid in the vessel is surrounded by a constant temperature enclosure, unlike in the method of mixtures.

NB:

- Steady state conditions are achieved when the thermometer readings stop fluctuating.
- Readings are only taken at steady state conditions because at these states the apparatus does not extract heat from the circuit.
- The p.d across the heater is accurately measured by potential meter because it is very sensitive to small voltages.
- Temperature is accurately measured using platinum wire thermometer since its resistance varies linearly with temperature.

Examples

1. A metal block of heat capacity $36.0J^oC^{-1}$ at 700^oC is plugged into an insulated beaker containing 200g of water at 18^oC. The block and the water eventually attain the same temperature of θ^oC. Assuming no heat loss to the surrounding or used to heat the beaker, find θ (s.h.c of water $4200Jkg^{-1}K^{-1}$)

Solutions:

Heat lost by the block=Heat gained by water
Since 1^oC change is equivalent to 1K change
$36x\,(70-\theta) = 200x10^{-3}x4200x\,(\theta-25)$
From which $\boldsymbol{\theta}$ =20.14℃.

2) A liquid of mass 200g in a calorimeter of heat capacity $500JK^{-1}$ is heated such that its temperature changes from 25^oC to 50^oC. Find the specific heat capacity of the liquid if the heat supplied was 14000J

Solutions:

Heat supplied = Heat gained by the liquid +Heat gained by the calorimeter
$14000 = 0.2xcx\,(50-25) + 500x\,(50-25)$

$c = 300Jkg^{-1}K^{-1}$

3) Use the information given below to calculate the specific heat capacity of the solid by the method of mixture.

Initial temperature of solid = 80°C, mass of solid = 600g, mass of liquid = 80g, H.C of calorimeter = $500JK^{-1}$

Initial temperature of liquid = 25°C, s.h.c of liquid = $2200Jkg^{-1}K^{-1}$

Final temperature of the mixture = 35°C.

Solutions:

Heat lost by the solid = Heat gained by the solid + Heat gained by the calorimeter.

$600x10^{-3}xc_sx\ (80-35) = (500+2200x80x10^{-3})(35-25)$

$c_s = 250.4Jkg^{-1}K^{-1}$

7) A substance of mass 1.0kg is supplied with 2800J of energy and it's temperature was raised by 5°C. Find its specific heat capacity.

Solutions:

Heat supplied = Heat which raises the temperature of the substance

$2800 = 1xc_sx5$

$c_s = 560Jkg^{-1}K^{-1}$

8) A mass of 1000g is forced through a tube 2m long by a steady force of 500N. Find the rise in its temperature of the substance when it is fully forced into the tube.

Work is done to raise the temperature of the substance

Force x distance = $mc_s\Delta\theta$

$500x2 = \frac{1x2800x\Delta\theta}{5}$, $\Delta\theta \approx 1.79$℃.

9) In a continuous flow method, the following results were obtained

Experiment	1	2
P.V/V	6	4.5
Current/A	2	1.5
In let temperature/K	38	38
Out let temperature/K	42	42
Time/s	60	180
Mass collected/g	42.3	70.3

Calculate the specific heat capacity of the liquid and the rate of loss of heat to the surrounding.

Solutions:

i) Rate of flow $m_1 = \frac{42.3 \times 10^{-3}}{60} = 7.05 \times 10^{-4} kgs^{-1}$

Rate of flow $m_2 = \frac{70.3 \times 10^{-3}}{180} = 3.91 \times 10^{-4} kgs^{-1}$

$$c_l = \frac{I_1V_1 - I_2V_2}{(m_1 - m_2)(\theta_2 - \theta_1)} = \frac{6*2 - 4.5*1.5}{(7.05 - 3.91)(42 - 38)*10^{-4}} = 4179Jkg^{-1}K^{-1}$$

ii) From $I_1V_1 = m_1c_l(\theta_2 - \theta_1) + h$

$$h = 6*2 - 7.05*10^{-4}*4179(42 - 38) = 0.215J/s$$

10) Using water which enters at 18°C and leaves at 22°C, the rate of flow is 20gmin^{-1}, current in the heating coil is 2.3A and the p.d across is 3.3V. Now using oil which flows in and out at the same temperature as water, the rate of flow is 70g/min and the current is 2.7A with p.d of 3.9V. If the s.h.c of water is 4200Jkg^{-1}K^{-1}, find;

i) The rate of heat loss from the apparatus, briefly explain your method.
ii) The specific heat capacity of the oil.

Solutions:

i) We assume steady state conditions such that the heat supplied is equal to the heat absorbed by the circulating liquid to upset the heat losses to the surrounding.

For experiment 1,

$$I_1V_1 = m_1c(\theta_2 - \theta_1) + h$$

$$2.3*3.3 = 0.33*10^{-3}*4200*4 + h$$

$$\Rightarrow h = 1.99W$$

iii) Since the mean temperature is the same in both cases, then **h** is the same so that

For experiment 2,

$$I_2V_2 = m_2c^1(\theta_2 - \theta_1) + h$$

$$c^1 = \frac{(3.9 \times 2.7 - 1.99)}{1.77 \times 10^{-3} \times 4} = 1830Jkg^{-1}K^{-1}$$

b) Explain how else you would determine c^1 if that of water was not known.

Solutions:

Carrying out two experiments with different rates of flow but in each, current and p.d are set such that the same steady temperatures are obtained. The two experiments would yield

$c^1 = \frac{(I_1V_1 - I_2V_2)t}{(m_1 - m_2)(\theta_2 - \theta_1)}$.

11) A car of weight 15KN is moving uniformly at 12m/s down a 1 in 6 hill with the engine switched off. If the break pedals have a mass of 30Kg and specific heat capacity of 0.40KJKg^{-1}K^{-1}, find the rate of increase in the pedal's temperature.

Solutions:

The car's rate of gain in Kinetic energy $=\frac{1}{2}m\left(\frac{v}{t}\right)v$ = 1/2xDriving force × v = 1/2xmg×sinθ×v

0.5x15x1000x1/6x12 =15W

Assuming this rate of gain in kinetic energy to be converted into heat absorbed by the pedals as the car gets on the leveled ground; 147150=30x400$x\frac{\Delta\theta}{\Delta t}$ from which $\frac{\Delta\theta}{\Delta t}$=12.2625Ks^{-1}

12) Two identical conductors each of heat capacity 12JK^{-1} cools from 325K. One holds to $8\times10^{-5}m^3$ of water and takes 150s to cool from 325K to 320K and the other holds an equal volume of unknown liquid which takes 50s to cool over the same range.

If the density of the liquid is $8.0\times10^2kgm^{-3}$, what is the average specific heat capacity of the liquid over 325K to 320K?

Solutions:

Since the containers are identical, the rate of heat loss from each is the same i.e.

Rate of heat lost by container A = Rate of heat lost by container B

$$\Rightarrow (m_w c_w + c)\frac{\theta_1 - \theta_2}{t_w} = (m_1 c_1 + c)\frac{\theta_1 - \theta_2}{t_1}$$

$$(8*10^{-5}*1000*4200+12)*\frac{5}{150} = (8*10^{-5}*800*c_l + 12)*\frac{5}{50}$$

$$\therefore c_l = 1625JKg^{-1}K^{-1}$$

13) Water flows at a steady rate of 6.0gs^{-1} through a continuous flow calorimeter. When the p.d across the coil is 11V and current is 5.0A. The difference between the inflow and outflow temperatures is 2.0K. When the flow changes to 2.0gs^{-1}, current is adjusted to 3.1A so as to produce the same temperature rise. Find

(i) The p.d across the heating coil, and hence the new input

(ii) The specific heat capacity of the water.

Solutions:

i) $V^1 = I^1R = \left(3.1*\frac{11}{5.0}\right)V = 6.82V$

The new input $= I^1V^1 = 3.1*6.82 = 21.1W$

$$(IV - IV^i) = \left(\frac{m}{t} - \frac{m^1}{t}\right)c\Delta\theta$$

ii) $5*11 - 3.1*6.82 = (6-2)*10^{-3}*c*2$

$$\Rightarrow c = 4238Jkg^{-1}K^{-1}$$

14) 50N on a cord is turned through 200 revolutions over a copper rod of diameter 25.0mm and mass 0.2kg using a motor. Calculate specific heat capacity of the copper rod if its temperature is raised by 10K.

Solutions:

Mechanical energy used = heat generated in the copper rod

Circumference x force x number of revolution made = $mc\Delta\theta$

50x (πx0.0250x200 =0.2xcx10

$c = 390.0Jkg^{-1}K^{-1}$.

2.0.3 Test 2

1) When water was passed through a continuous flow flask, a rise in temperature from 16°C to 20°C was recorded. The mass of water flowing was 100g in 1 minute. The P.d across the heating coil is 20V and current is 1.5A. Another liquid at 16°C was then passed through the flask and to get the same change in temperature, the P.d was changed to 13V and the current to 1.2A with 110g of the liquid flowing in 1 minute. Given that the s.h.c of water is $4200Jkg^{-1}K^{-1}$, calculate;

i) The rate of heat loss to the surrounding.
ii) The specific heat capacity of the liquid.

2) In a continuous flow method for determining the s.h.c of a liquid of density $800kgm^{-3}$, a volume of $3.6\times10^{-3}m^3$ of liquid flows through apparatus for 10 minutes. If energy is supplied to the heating coils at a rate of 44W, a steady temperature difference of 4K is obtained between the inlet and outlet temperatures. If the rate of flow is increased to $4.8\times10^{-3}m^3$ in 10 minutes, the electrical power required to maintain the same temperature difference is 58W. Find

i) The specific heat capacity of the liquid.
ii) The rate of heat loss to the surrounding.

3) In the Nernst calorimeter, the mass of the solid was 0.8kg and the temperature rise was 1.2K when a current of 3.0A with a P.d of 2.0V flows for 22s. When a current of 2.7A with P.d of 1.8V flows for the same time, the temperature rise was 0.97K. Find for this temperature rise;

i) The specific heat capacity of the solid.
ii) The power loss.

4) A drill using a current of 2.0A when connected to 240V mains supply makes a hole in a piece of Iron of mass 0.80kg. Find the temperature rise in 20s assuming 60% of electrical energy is converted into Iron's internal energy (heat) (specific heat capacity of Iron is $460Jkg^{-1}K^{-1}$)

5 (a) In continuous flow experiment explain how you would achieve the following.

(i) Steady state conditions.
(ii) Accurately Measure temperature difference between in flow and out flow liquid
(iii) Accurately measure P.d across and current through the coil.

(b) Explain why in continuous flow experiment, readings are only recorded when conditions are steady

(d) (i) Define cooling correction.

(ii) State the precautions taken when performing continuous flow experiment.

(e) Explain how cooling correction may be estimated in the determination of heat capacity of a poor conductor of heat by method of mixtures.

6 (a) (i) Define specific heat capacity of a substance.

(ii) State how heat losses are minimized in the colorimeter.

(b) Describe how you would determine the specific heat capacity of a liquid by the continuous flow method.

c) Give two advantages and two disadvantages of continuous flow method.

(d) In a continuous flow method for measuring specific heat capacity of a liquid, $3.6 \times 10^{-3} m^3$ of a liquid flows through the apparatus in 10 minutes, when the electrical energy is supplied to the heating coil at the rate of 44w a stead difference of 4k is obtained between the temperature of the out flowing and inflowing liquid. When the flow rate is increased to $4.8 \times 10^{-3} m^3$ of the liquid in 10 minutes, the electrical power required to maintain the temperature difference is 58w. Find the;

i) Specific hat capacity of the liquid.

ii) Rate of heat loss to the surrounding.

[-Density of liquid = $800 kgm^{-3}$]

7 (a) Describe an experiment to determine s.h.c of a solid by electrical method

(b) A metal container of heat capacity $200 JK^{-1}$ holds 0.15kg of liquid. An immersion heater rated at 3.0A and 12V was placed into the container for some time.

Temp/℃	15.0	20.0	24.7	29.3	33.9	32.8	31.7	30.6
Time/min	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0

Plot a suitable graph to determine the cooling correction, hence find the s.h.c of the liquid.

8) (a) 7g of a liquid is placed in a copper calorimeter of mass 50g at 16.8℃. An immersion heater of negligible heat capacity rated 1.8A and 7.0V was used to heat the liquid for 4minutes such that its temperature rises by 11.9℃. Subsequently temperature dropped to26.1℃ during the next two minutes from when heating end. Calculate s.h.c of copper in $Jg^{-1}K^{-1}$ given that that of the liquid is $4.2 Jg^{-1}K^{-1}$.

(b) A copper calorimeter of mass 50g and s.h.c $400 Jkg^{-1}K^{-1}$ contains a liquid of mass 100g at 19.3℃ . Temperature of the mixture is raised to 28.7℃ by an immersion heater rated 1.5A and 11.5V with negligible thermal heat capacity for 5.4minutes. Subsequently temperature falls to 27.9℃ after 2.5minute since when heating was stopped. Caliculate;

(i) Cooling correction.

(ii) Corrected maximum temperature

(iii) Corrected temperature rise
(iv) Specific heat capacity of the liquid
(v) Heat lost during heating
(vi) Rate of heat loss during heating the mixture
Ans(0.864℃, 29.564℃, 10.264℃, 470.7J and 5245Jkg^{-1}K^{-1}, 1.45Js^{-1})

9(a) A copper block of mass 250g with s.h.c of 410Jkg^{-1}K^{-1} is heated to 145℃ and then dropped into a copper calorimeter of mass 300g which contains 250m^3 of pure water at 20℃. Calculate the new equilibrium temperature of the liquid and sketch a graph to show variation of temperature with time for;

(i) Copper block
(ii) Water
(iii) Both copper block and water on the axes.

(b) A copper calorimeter of mass 50g and s.h.c 400Jkg^{-1}K^{-1} contains a liquid of mass 100g at 19.3℃ . Temperature of the mixture is raised to 28.7℃ by an immersion heater rated 1.5A and 11.5V with negligible thermal heat capacity for 5.4minutes. Subsequently temperature falls to 27.9℃ after 2.5minute since when heating was stopped. Taking the rate of heat loss during heating process to be 1.45Js^{-1}, calculate;
(i) Cooling correction
(ii) Corrected maximum temperature
(iii) Corrected temperature rise
(iv) Heat lost during heating
(v) Specific heat capacity of the liquid
Ans (0.864℃, 29.564℃, 10.264℃, 470.7J and 5245Jkg^{-1}K^{-1})

10) a) The same amount of heat used to raise temperature of120g of water from 25℃ to 60℃ is used to heat a metal rod of 1.7kg and s.h.c 300Jkg^{-1}K^{-1} at 20℃. Find the temperature to which the solid is raised.

b) Hot water at 90℃ and cold water at 10.6℃ are into a tank of capacity 1.5m^3 at rates of 0.0065m^{3}perminute and V respectively. When the tank is full, temperature of water was 41.7℃. Determine the duration of filling the tank if the tank has a negligible thermal heat capacity.

c) In a continuous flow experiment the first readings were; 6.0V, 2.1A, θ_1=17.0℃, θ_2=22.0℃ 35g per minute followed by 4.0V, 1.4A, θ_1=17.0℃, θ_2=22.0℃, and 15gper minute. Calculate s.h.c of the liquid and the rate of heat loss to the external environment.

11) (a) 21.0g of a liquid at 60.0℃ is mixed with 100g 0f water contained in a metal calorimeter of mass 70.0g and s.h.c of 400Jkg^{-1}K^{-1} at 12.5℃. calculate the equilibrium temperature of the mixture given that; s.h.c of water is 4200Jkg^{-1}K^{-1} and that of the liquid is 4000Jkg^{-1}K^{-1}.

b) 90g of a liquid were placed in a 40g copper calorimeter of s.h.c 400Jkg^{-1}K^{-1} and heated for one minute and 40s using 14V and 3.0A. Temperature rose from 10.0℃ to 30.1℃ and subsequently fell to 28.8˚C during the next 50s from when heating was stopped. Calculate s.h.c of the liquid if thermal capacity of the heater is negligible.

12) (a) A metal block of heat capacity 360JK^{-1} at 70℃ is dropped into an insulated copper calorimeter of s.h.c 400Jkg^{-1}K^{-1} and mass of 500g containing 200g of water of s.h.c 4200Jkg^{-1}K^{-1} at a common temperature of 18.0℃. Calculate the final temperature of the mixture and mention any assumption made.

(b) A solid of mass 600g at 80.0℃ and s.h.c of 250.4Jkg^{-1}K^{-1} is dropped into a liquid of 80.0g in a calorimeter of heat capacity 500JK^{-1} such that temperature of the liquid is raised from 25.0 ℃ to 56.5℃. Calculate the s.h.c of the liquid.

c) An electrical heater rated 6.0V and 3.50A is used raise temperature of an insulated copper block of s.h.c of 400Jkg^{-1}K^{-1} from 20.0℃ to 41.9℃ for 2.0minutes. Calculate mass of copper used.

(d) How long would it take a heater rated 100W to raise temperature of a liquid of heat capacity 1800JK^{-1} by 11.5℃ contained in a copper calorimeter of mass 800g and s.h.c 400Jkg^{-1}K^{-1} by such a heater. State major assumptions mad

Latent heat

Related concepts

(i) Latent Heat is the amount of heat required to change the state of a substance at constant temperature.

(ii) Specific latent heat is the amount of heat required to change a physical state of **1kg** mass of a substance at constant temperature.
(iii) Latent heat of fusion(Q_f) is the amount of heat required to change any mass of a solid at its melting point into a liquid at the same temperature or a any mass of liquid to solid at constant temperature.
(iv) Specific latent heat of fusion (L_f) is the amount of heat required to change a **1kg** mass of a solid at its melting point into a liquid at the same temperature or a liquid to solid at constant temperature i.e. $Q_f = mL_f$.
(v) Latent heat of vaporization (Q_V) is the amount of heat required to change any mass of liquid at its boiling point to vapor at the same temperature or any mass of vapor to liquid at constant temperature i.e. $Q_v = mL_v$.

(vi) Specific latent heat of vaporization (L_v) is the amount of heat required to change a **1kg** mass of liquid at its boiling point to vapor at the same temperature or vapor to liquid at constant temperature.

Kinetic theory explanation for latent heat of vaporization and latent heat of fusion

In solids, molecules are in constant vibrations about their mean positions yet in liquids molecules are relatively free to move.
Energy is therefore required to weaken the mutual attractions of solid molecules to let them free to move as the case in liquids. This absorbed energy is called latent heat of fusion of solid to liquid.
In vapors, molecules are completely free from mutual attractions of their neighbours thus occupying a lager volume to that a liquid occupies.
Energy is required to completely break the mutual attractions of the liquid molecules and to enable its expansion against atmospheric pressure. This absorbed energy is called latent heat of vaporization of liquid to vapor.

NB:

Latent heat of vaporization is greater than latent heat of fusion of the same substance because for vaporization to occur bonds of that substance must be completely broken yet for fusion to occur bonds are just weakened.

NB:

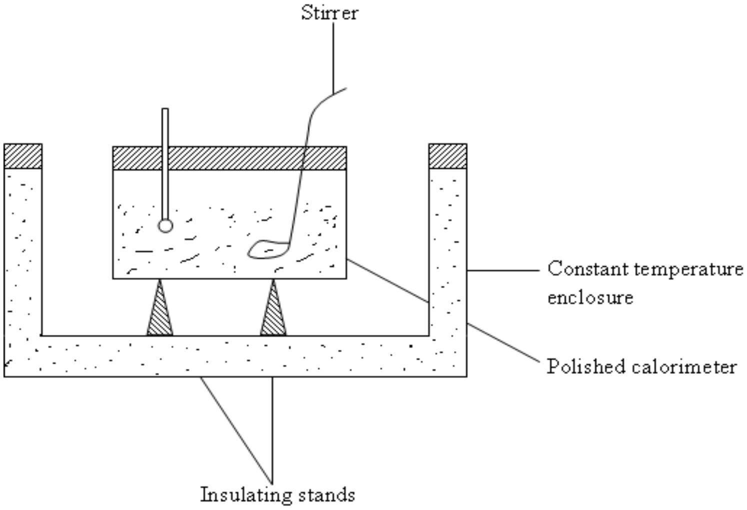
The specific latent heat of vaporization is different at different temperetures from that at boiling point. Forexample at 20^0C, the molecules of the liquid are close together than at boiling point. The intermolecular forces at 20^0C are stronger than at boiling point. More energy is therefore required to separate/ break the bonds at 20^0C than that needed at boiling point.

Question:

1. Explain why specific latent heat of vapourisation is greater than specific latent heat of fusion.
2. Explain why specific latent heat of vapourisatins is higher at 20^0C than at boiling point.

Determining specific latent heat of fusion

By method of mixtures



A known mass $\mathbf{m_w}$ of water of known specific heat capacity $\mathbf{c_w}$, is heated up to a temperature θ_1 and put into a polished calorimeter of known heat capacity $\mathbf{C_c}$ as shown above.
A known mass of pure ice $\mathbf{m_i}$ is dropped into the calorimeter and the mixture is stirred until equilibrium temperature θ_2 of the mixture is attained. This temperature θ_2 is read from the thermometer and recorded. Neglecting heat losses to the surrounding implies that;

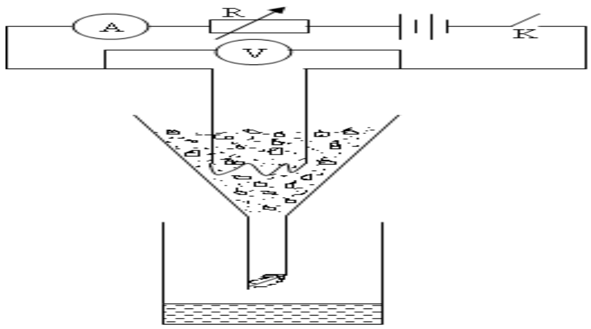
$$(Heat lost by calorimeter and water in it) = (Heat which melts ice) + (Heat which raises its temperature from\ 0^{\circ}C to\ \theta_2)$$

$$(m_w c_w + C_c)(\theta_1 - \theta_2) = m_i L_f + m_i c_w \theta_2$$

Main sources of errors in this experiment.

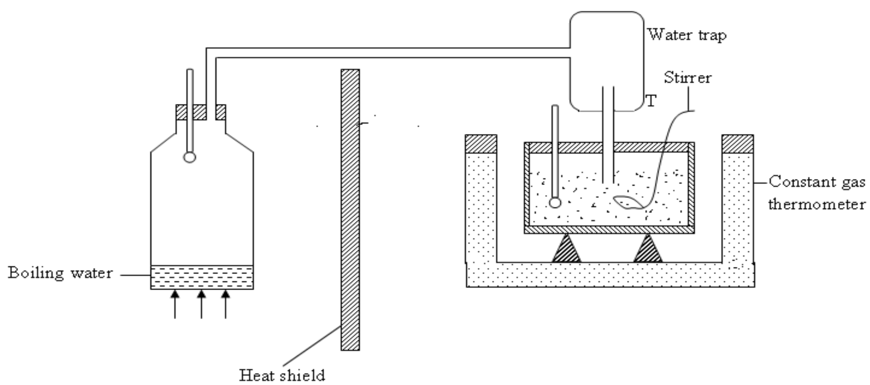
- Ice may be wet but this is minimized by pressing ice between bloating paper to dry it first.
- Condensation of dew on the outside surfaces of the calorimeter may lead to evolution of Latent heat from the calorimeter.

Electrical method



Dry pure ice is put into the funnel such that it submerges the heater.
With R set, K is closed and the stop clock is started.
A beaker of known mass is used to collect the melting ice and after a given time interval t, values of I, V are read and recorded.
K is opened and the mass of melted ice m_i is determined by a beam balance.
Assuming no heat losses to the surrounding, $IVt = m_i L_f$
N.B: If heat loss to the surrounding (h) is not neglected, then;
$IVt = m_i L_f + h$ ……… (i)
The experiment is repeated with new dry ice and R set to a new value R^1 within the same time interval **t** of heating.
$I_1V_1t = m_i^1 L_f + h$ ………(ii)
Combining equation (i) and (ii) yields $L_f = \frac{(IV - I_1V_1)t}{(m_1 - m_1^1)}$.

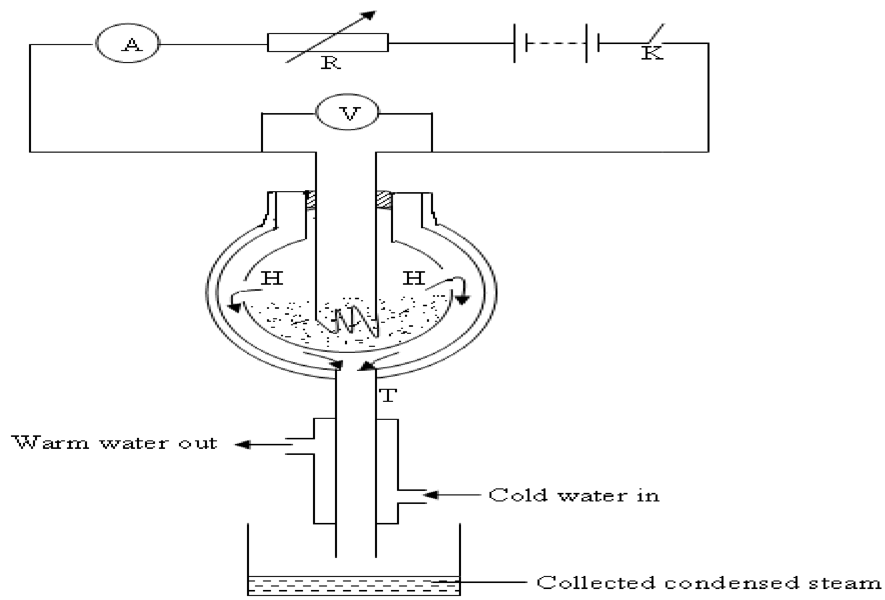
Determining latent heat of vaporization
By method of mixtures



The mass m_1 of the calorimeter and the waterin it is measured and recorded.
The temperature θ_1 of the water and calorimeter is read and recorded
Steam from boiling water is then passed into the calorimetr containing water, through the steam trap.

After a measurable temperature range, the final temperature θ_2 of the mixture is read and recorded.
The new mass m_2 of the calorimeter and its content is determined and mass m_s of the condensed steam is calculated from $m_s = m_2 - m_1$.
The mass m_c of the empty calorimetr is also found.
Heat lost by steam = heat absorbed by (water+ calorimeter) i.e.
$m_sL_v+m_sc_w(100-\theta_2) = m_cc_c(\theta_2-\theta_1)+m_wc_w(\theta_2-\theta_1)$, from which L_v can be calculated.

Electrical method.



The specimen liquid at its boiling point vaporizes when K is closed.
Vapor passes through holes H down-tube T and is condensed by the water jacket (condenser).
At steady state i.e. (when the liquid is boiling at steady temperature) the liquid emerging from T is collected in a beaker of known mass for a measured time t.
The mass **m_s** of condensed steam is determined by weighing the collecting dish on abeam balance such that ms = mass of (condensed steam+dish) –mass of the empty dish.
The readings I_1, V_1 of the ammeter and voltmeter are read and recorded such that $I_1V_1t = m_sL_v + h$ …………… (i) Where **h** is heat lost to the surroundings in time t.
The experiment is repeated with R set to a new value R^1 and heating done in the same time interval so that at steady state, $I_2V_2t = m_s^1L_v + h$ …………….. (ii)
Combining equations (i) and (ii) yields $L_v = \frac{(I_1V_1 - I_2V_2)t}{(m_s - m_s^1)}$.

Examples

1) Water in a vacuum flask is boiled steadily by a coil of wire immersed in it. When the p.d across the coil is 5.25V and the current through it is 2.58A, 6.85g of water evaporates in 20 minutes. When the p.d and current are maintained at 3.20V and 1.57A respectively, 2.38g of water evaporates in 20 minutes other conditions being the same. Find Latent heat of vaporization of water and the rate of heat loss.

Solutions:

From $L_v = \frac{(I_1V_1 - I_2V_2)t}{m_1 - m_2} = \frac{(5.25x2.58 - 3.2x1.57)x20x60}{(6.85-2.38)x10^{-3}}$ =2.29x10^{6}Jkg^{-1}

h= $I_1V_1 - m'_1L_v$ =5.25x2.58-$\frac{6.85x10^{-3}x2.29x10^6}{20x60}$ =0.4729W

2) An alluminium pail of mass 1.5kg contains 1kg of water and 2kg of ice at 0^{o}C. If 3kg of water at 70^{o}C is poured into this pail, calculate the final temperature of the pail plus water (s.h.c of Aluminium=910Jkg^{-1}K^{-1}, s.h.c. of water=4200Jkg^{-1}K^{-1} and L_f of ice=33600Jkg^{-1}).

Solutions:

Heat lost by water at 70^{o}C = Heat gained by 1.5kg of water + Heat gained by calorimeter + Heat which melts ice + Heat which raises the temperature of the ice to θ_f.

$$3*4200*(70-\theta_f) = (1.5*910+1*4200)(\theta_f - 0) + (2*3.36*10^5 + 2*4200)(\theta_f - 0)$$

$$\Rightarrow \theta_f = 7.9^o C$$

3) A stream of electrons each of mass 9.0×10^{-31}kg and velocity 2.0×10^{7}m/s strikes 5×10^{-4}kg of silver initially at 20^{o}C mounted in a vacuum. Assuming 10^6 electrons strike the silver per second and that energy is converted into heat which is retained by the silver, calculate the time taken for the silver to melt (s.h.c. of silver between 20^{o}C and 960^{o}C is 2.35×10^{2}Jkg^{-1}K^{-1}, melting point of silver=960^{o}C and specific latent heat of fusion of silver=1.09×10^4 Jkg^{-1}).

Solutions:

Heat lost by hitting electrons =$(Heatwhichraisessilver'stempfrom\ 20^\circ Cto\ 960^\circ C) + (Heatwhichmeltssilverat\ 960^\circ C)$

Kinetic energy of electrons in a unit time, (k.e)'= 0.5x9.0x10^{-31}x (2.0x10^7)^{2}x10^6 =1.8Js^{-1}

Therefore, k.e = (k.e)' × time 1.8tJ

Heat required to melt ice =5x10^{-4}x2.33x10^{2}x (960-20) + (5x10^{-4}x1.09x10^4) =115.9J

Then for the steady state, 1.8t=115.9 $\Rightarrow t = 64.4s$

4) Calculate the external work done and internal energy gained when 200g of water at 100℃ with density 1.43kgm^{-3} at standard atmospheric pressure of 1.01x10^{5}pa is converted to steam given that L_v =2.26X10^{6}Jkg^{-1}.

Solutions:

Work done = Pressure x Volume =$\frac{Pm}{\rho}$ =$\frac{1.01x10^5x200x10^{-3}}{1.43}$ ≈14,125.87J.

Internal energy = mL_v = 200x10^{-3}x2.26x10^6 =452,000J.

5) An electric heater rated 500W is immersed in 4kg of water at 27℃ contained in a copper vessel of s.h.c of $400Jkg^{-1}K^{-1}$ and mass 0.584kg with negligible heat losses to surroundings. Taking $L_v = 2.26x10^6 Jkg^{-1}$ determine;

(i) How long it takes to heat water to 100℃.

(ii) How long it takes to boil off the whole water.

(iii) How much it costs to achieve (i) and (ii) if 1kW costs 7.0/=.

Solutions:

(i) **E**lectrical energy = Heat which raises temperature of (water+ calorimeter) from 27℃ to 100℃.

$500t_1 = 400x0.584x\ (100\text{-}27) + 4x4200x\ (100\text{-}27)$

$t_1 = 0.69hrs.$

(ii) Electrical energy = Heat which vaporizes the liquid.

$500t_2 = 4x2.26x10^6$

$t_2 = 5.02hrs.$

(iii) Total time 5.71hrs.

Total power = 500x5.71 =2.855kWhr.

1kWhr costs 7.0/=

2.855kWhr costs 2.855x7 = 19.985/=.

6. An electrical heater of 2.5kW is used to heat 2litres of water in a kettle of heat capacity 400Jkg-1. If the initial temperature of water is 24^0C, and neglecting heat loses to the surroundings, find:

i) how long it will take to heat the water to boiling point.(4.5min)

ii) The mass of water boiled off in 5 minutes, if heating started from 24^0C.(0.038kg)

Exercise

1) Ice at 0^oC is added to 200g of water at 70^oC in a vacuum flask. When 50g of ice has been added and has all melted, temperature of the flask and its contents is 40^oC. When a further 80g of ice has been added and melted, temperatures further reduces to 10^oC. Find the specific latent heat of fusion of ice and heat capacity of the flask neglecting heat loses to the surroundings.

2) In an experiment to determine the latent heat of vaporization, the following data was obtained.

Ammeter reading/A	1.00	2.00	2.30	2.63
Voltmeter reading/V	5.0	5.4	7.0	8.0
Mass of liquid vaporized in 100s/g	0.8	3.6	6.0	8.4

Plot a suitable graph and use it to determine latent heat of vaporization and the rate of heat loss to the surrounding (Hint: $IV = m'_1L_v + h$).

3) A copper calorimeter of mass 0.4kg contains 0.2kg of water at 25^oC. Crushed ice at 0^oC is introduced into the calorimeter as the mixture is continuously stirred. When all the ice

has melted, the temperature of the calorimeter and its contents is found to be 20°C. Find the mass of ice added and state the assumptions made (specific heat capacity of Cu=$400Jkg^{-1}K^{-1}$, specific latent heat of fusion of ice is $3.4\times10^5Jkg^{-1}$).

4) (a) Explain the following observations;

(i) A volatile liquid like mentholated spirit or ether cause a more noticeable coldness if poured on a human body than a less volatile liquid like water.

(ii) Latent heat of vaporization is greater than latent heat of fusion of the same substance.

(iii) A scar is more dangerous than a burn.

(iv) Water is preferred to any other liquid to be used in cooling systems of vehicles i.e. car radiators.

(b) Use kinetic theory to account for the occurrence of latent heat of fusion and latent heat of vaporization

5) (a) If melting point of lead is 327℃, with s.h.c of $140Jkg^{-1}K^{-1}$ and specific latent heat of fusion 2.7x105Jkg^{-1}. Determine the amount of heat required to melt 200g of lead initially at;

(i) 27℃

(ii) 327℃

(b) A calorimeter of mass 35.0g and s.h.c $840Jkg^{-1}K^{-1}$ contains 143.0g of water at 7.0℃. Dry steam at 100℃ is bubbled through water in the calorimeter until when temperature rises to 29℃. If mass of steam condensed is 5.8g, determine specific latent heat of fusion of steam.

c) The cooling system of a refrigerator extracts 0.7kW of heat. How long will it take to convert 500g of water at 20℃ to solid ice at 0℃ given that s.h.c of water is $4200Jkg^{-1}K^{-1}$ and latent heat of fusion of ice is $3.46x10^4J$?

6) (a) A copper container of heat capacity $60JK^{-1}$ contains 0.05kg of pure water at 23.0℃. Dry pure steam is passed through into the container and rises to 61.8℃. Calculate the mass of stead condensed given that $\mathbf{L_v}$of steam is $2.26x10^6Jkg^{-1}$ neglecting heat losses to the surroundings.

(b) A copper can of mass 0.25kg and s.h.c of $390Jkg^{-1}K^{-1}$ contains 20.0g of water at 11.6℃. The can with its contents is placed in a refrigerator. Calculate the quantity of heat the refrigerator extracts to reduce the temperature of the can to -2.1℃ given that specific latent heat capacity of fusion of ice is $3.6x10^5Jkg^{-1}$ and s.h.c of ice is $2150Jkg^{-1}K^{-1}$.

c) Describe a simple experiment to determine **latent heat of fusion** of ice by electrical method.

7) (a) With aid of a labeled diagram, describe how you would determine latent heat of vaporization of pure water electrically.

(b) 1.0kg of pure steam is converted to vapor at standard atmospheric pressure and temperature (s.t.p). Determine;

(i) External work done by steam.

(ii) Internal gained by steam

c) A piece of ice of mass 0.0006kg at 272K is dropped into a vacuum flask containing $8.0x10^{-4}m^3$ of nitrogen of density $1.25kgm^{-3}$ which melts at 77K. Determine s.h.c of ice between 272K and 77K given that $\mathbf{L_v}$of nitrogen is $2.13x105Jkg^{-1}$.

8) In an experiment to determine specific latent heat of vaporization of alcohol using a self-jacketing vaporizer the following results were obtained.

Experiment 1

V_1=7.00V, I_1=2.60A, m_1 =5.80g collected in300s.

Experiment 2

V_2 = 10.0V, I_2 =3.60A, m_2 =11.90g collected in 300s.

Calculate;

(i) Specific latent heat of vaporization

(ii) Average rate of heat loss

(iii) Rate at which the heater should be rated such that the rate of evaporation is $1,5gmin^{-1}$.

Boiling, Melting, Evaporation and Cooling

Boiling/vaporization:

This is defined as the process by which a liquid turns to vapor at constant temperature (boiling point) i.e.

Boiling point is the constant temperature of a liquid at which its saturated vapor pressure is equal to external atmospheric pressure.

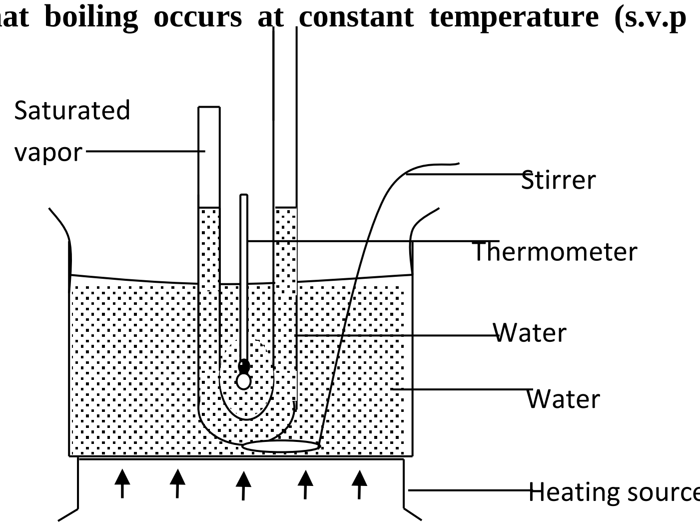
Steam point is the constant temperature at which (pure water ↔vapor) at 760mmHg.

It is 100℃.

NB:

It should be noted that boiling of any liquid occurs only when its saturated vapor pressure equalizes with external pressure implying that when a liquid is boiling, there is change of state thus occurring constant temperature called boiling point.

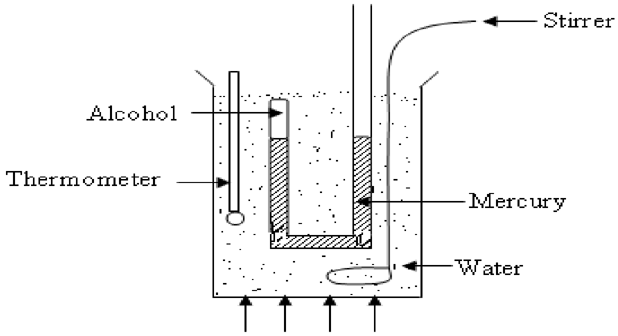
Experiment to show that boiling occurs at constant temperature (s.v.p = external pressure).



Water is trapped in a closed end of J-shaped tube and the tube is placed in a beaker containing water being heated from the base as it is stirred to ensure uniform distribution of heat throughout the liquid.
When water in the beaker starts boiling, its vapor escapes and exerts pressure on water in the open limb of the J- shaped tube.
At this point the thermometer reading remains constant and water in the J- shaped tube levels up indicating that saturated vapor pressure is equal to external pressure.

Determining boiling point of a liquid whose Boiling point is less than 100^{o}C e.g. Alcohol

The liquid whose boiling point is to be determined is trapped my mercury in the closed end of a U-tube.
The tube is placed in a large beaker of water together with a thermometer.
Water is slowly heated as continuous stirring is done to ensure uniform distribution of heat.
As temperature rises, the liquid vaporizes and exerts pressure on mercury in the open limb until when mercury in both limbs levels up i.e. (Vapor pressure = Atmospheric pressure).
The reading of the thermometer is taken to be the boiling point of the liquid.



Cooking at a higher altitude

Since boiling only occurs when vapor pressure is equal to the external atmospheric pressure, and that boiling point decreases with decrease in atmospheric pressure, yet atmospheric pressure reduces with altitude, then cooking is slower on a higher altitude than on a leveled ground though occurs faster.

Pressure cooker

Cooking with a pressure cooker is faster than ordinary cooking because most of its top surface is covered leaving just a small opening to let out vapor.

This covering reduces the space of escape for vapor molecules which increase the pressure inside due to random collisions of vapor molecules thus raising the boiling point to about 120^oC, hence faster cooking due too much heat.

NB:

- Water can be made to boil at temperature less than 100℃ by boiling it at higher altitude or boiling it when it is free of impurities.
- Addition of impurities raise the boiling point of a liquid since impurities absorb some of the supplied heat making the liquid to boil at a higher temperature than its normal boiling point thus faster cooking.

Melting

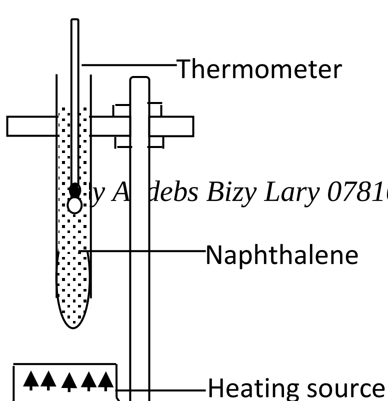
This is defined as the process by which a solid turns to liquid at constant temperature called melting point i.e.

Melting point is constant temperature at which a solid substance liquidizes at constant atmospheric pressure.

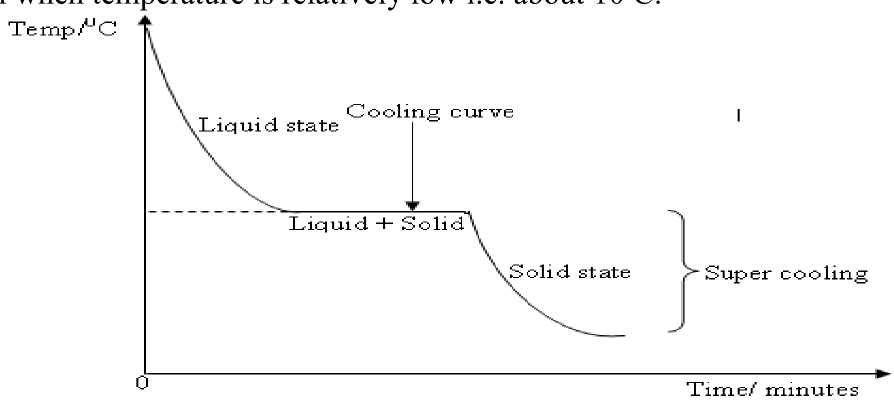
Freezing point is constant temperature at which a molten substance solidifies constant atmospheric pressure.

Ice point is a constant temperature at which (pure solid ice ↔ pure liquid ice) at 760mmHg. It is 0℃.

Determining melting and freezing points of naphthalene



A test tube containing solid naphthalene with a thermometer is held vertically using a retort stand.
It is heated gently until when naphthalene has **jus**t completely melted
At this instant, the reading of the thermometer is recorded as the **melting point** of naphthalene.
For determining the its freezing point, heating is continued until when temperature of melted naphthalene is relatively high i.e. about 110℃
Heating is stopped and readings of thermometer are read and recorded after every one minute until when temperature is relatively low i.e. about 10℃.



Freezing point is the temperature value read from the graph corresponding to its flat portion.

NB:

- Skaters slide easily over ice because the work done against friction is transferred into internal energy which makes ice to melt forming a thin film of water between the blades of the skate and ice which eases the sliding.
- A weighed wire passes through a block of ice with out cutting it into two pieces because increased pressure due to weights on the wire lower the melting point of ice as water no longer under pressure refreezes as it gives out latent heat.

- Impurities like salt lower the melting point of a solid e.g. freezing point of pure ice is 0℃ but that for impure ice is less than 0℃.

Evaporation

This is the process by which a liquid turns to vapor and molecules leave the liquid surface. It takes place at all temperatures but it is greatest when the liquid is at its boiling point.
The rate of evaporation of a liquid is increased by;

- Increasing the surface area of the liquid e.g. same amount of water put in a basin and cup exposed to the same drought, one in a basin reduces faster than that in a cup.
- Increasing the temperature of the liquid since increase in temperature directly increases the average kinetic energy of the molecules escaping.
- Providing drought which removes the vapor molecules from the liquid surface before returning to it e.g. water exposed to direct sunshine evaporates faster than that under a shade.
- Reducing the pressure of the air above the liquid surface (atmospheric pressure) e.g. evaporation is faster on a mountain than on a leveled ground.

Comparison of boiling and evaporation

Evaporation.	**Boiling**.
Increases with increase in heat.	Independent of heat supplied.
Occurs on the surface of the liquid.	Occurs throughout the liquid.
It doe not necessary need saturated vapor pressure to be equal to atmospheric pressure.	Occurs when saturated vapor pressure is equal to atmospheric pressure.
It is invisible.	It involves formation of visible bubbles
Occurs at all temperatures	Occurs at constant standard temperature.
Can be accelerated by providing drought to remove vapor molecules from the neighbor hood.	Drought does not affect boiling point o f a liquid.

Cooling

This is defined as the continuous fall of temperature of a body placed in drought until when it attains an equilibrium state.

Kinetic theory of cooling by evaporation

Evaporation occurs when the most energetic molecules at the liquid surface escape. The molecules that remain are those with with low kinetic energy. Since mean kinetic energy of the molecules is directly proportional to the absolute temperature, the liquid cools.

Factors affecting the body's rate of cooling

- Temperature of the body's surface e.g. a body at higher temperature cools faster than an identical body at low temperature placed in the same environment.
- Temperature of the surrounding environment e.g. a body cools faster when placed in an environment of relatively low temperature compared to when is placed in an environment of relatively high temperature.
- The nature of the cooling body e.g. a good conductor of heat loose heat faster than a bad conductor of heat.
- Humidity of environment i.e. humidity is defined as the amount of water vapor in the atmosphere. The body's rate of cooling increases as humidity increase since much humidity corresponds to low temperature.
- The size of the body's enclosure e.g. a body cools faster when placed in a wider enclosure than when placed in a smaller enclosure due to difference in the ratios of volume to surface area.
- The nature of the body's enclosure e.g. a body cools faster when placed in an enclosure with openings than when placed in a closed enclosure.
- Size of the body e.g. smaller bodies cool faster than larger ones due to larger volume to surface area i.e. the larger the volume-surface area of a particular body, the faster is rate of cooling since the rate of temperature fall is inversely proportional to the body's volume i.e. from$\frac{dQ}{dt} = mc\frac{\Delta\theta}{\Delta t}$, we get that $\frac{\Delta\theta}{\Delta t} = \frac{\frac{dQ}{dt}}{Vc\rho}$$\Rightarrow$$\frac{\Delta\theta}{\Delta t}$**α**$\frac{1}{V}$.

Explain why a small body cools faster than a large one if they are made of the same material?

The rate of fall of temperature of a body is inversely proportional to its mass. Since the mass of the body is proportional to its volume and the rate of heat loss is proportional to surface area of the body then the rate of temperature fall is proportional to the ratio of surface area to volume of the body. The ratio of surface area to volume of a body is inversely proportional to linear dimensions, small bodies cool faster than big ones.

Newton's law of cooling

The law states that under conditions of forced convection, the rate of heat loss (temperature fall) of an object is directly proportional to the excess temperature over that of the surrounding i.e.$\frac{dQ}{dt}$=k($\boldsymbol{\theta}_s$-$\boldsymbol{\theta}_o$) or $\frac{d\theta}{dt}$=β($\boldsymbol{\theta}_s$-$\boldsymbol{\theta}_o$).

From the law above, $\frac{dQ}{dt} \propto (\theta - \theta_R) \Rightarrow \frac{dQ}{dt} = -k(\theta - \theta_R)$ but from $\frac{dQ}{dt} = mC\frac{d\theta}{dt}$

$\Rightarrow \frac{d\theta}{dt} = -k(\theta - \theta_R)$

Question: Use Newton's law of cooling to show that $\frac{d\theta}{dt} = -k(\theta - \theta_R)$

- **Verification.**

The room temperature θ_R is measured and recorded.

A liquid is heated to a high temperature and put in a calorimeter. The calorimeter is placed near a window. The temperature of the liquid is measured at equal time intervals and a cooling curve is plotted. The slopes of the curve at different temperatures are recorded. The slopes are plotted against corresponding excess temperatures over room temperature. A straight line graph shows that the rate of heat loss is proportional to the excsee temperature.

NB:
Using Newton's of cooling it can be shown that the rate of fall of body's temperature is directly proportional to the excess temperature **(θ_s-θ_t)** where; θ_s is temperature of the surrounding and θ_t is temperature of the body at time t.

Solutions:

By Newton's law of cooling $\frac{dQ}{dt}$=k(θ_s-θ_t)……..(i)

The quantity of heat Q = mc$\Delta\theta$ ……….(ii) where m is the body's mass and c is the body's specific heat capacity.

Deffericianting equation (ii) with respect to temperature yields $\frac{dQ}{dt}$=mc$\frac{d\theta}{dt}$…..(iii).

Comparing equations (i) and (ii) implies that k (θ_s-θ_t) = mc$\frac{d\theta}{dt}$ …..(iv)

Equation (iv) clearly shows that $\frac{d\theta}{dt}$=β(θ_s-θ_t) where β =$\frac{k}{mc}$.

Example

1. An electrical heater rated 520W is used to raise the temperature of 2.5kg of a liquid from 20^0C to 100^0C in 25 minutes. Given that the room temperature is 20^0C and the rate of heat loss at 100^0C is 16W, estimate the specific heat capacity of the liquid.

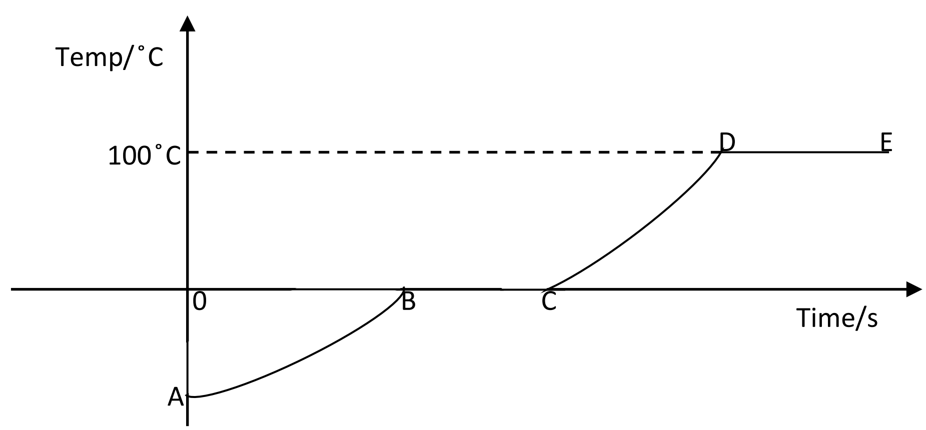
Solution

Average rate of heat loss = $\frac{0+16}{2} = 8W, total\ heat\ lost = 8x25x60 = 12000J$

Therefore, $520x25x60 = 2.5xCx(100-20) + 12000 \Rightarrow C = 3840Jkg^{-1}K^{-1}$

Cooling and Heating curves

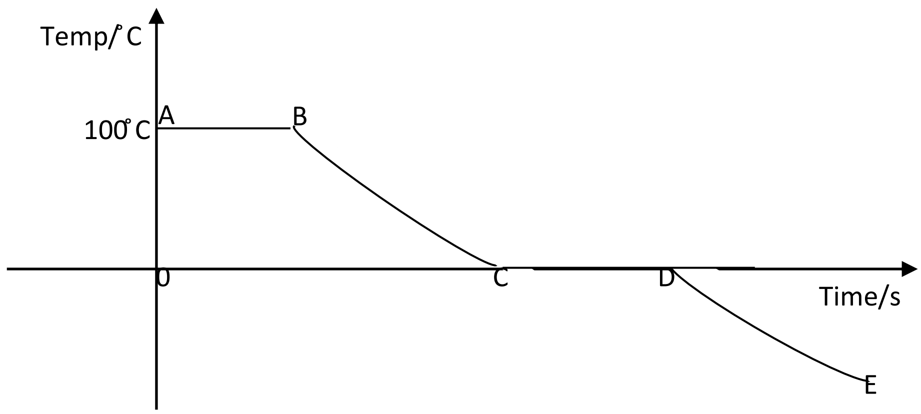
Heating ice which is at less than 0℃



Between:

- A to B, heat supplied is used to raise temperature of ice below 0℃ to 0℃.
- B to C, heat supplied is used to weaken the bonds of ice at 0℃ to water at 0℃ by a process of melting.
- C to D, heat supplied is used to raise temperature of water at 0℃ to 100℃.
- D to E, heat supplied is used to completely break the bonds of water at 100℃ to form vapor at 100℃ by process of vaporization.

Cooling steam below 0℃



Between;

- A to B, heat is lost as steam turns to liquid at 100℃ a process called condensation.
- B to C, heat is released so as to cool water at 100℃ to water at 0℃.
- C to D, heat is released so as to turn water at 0℃ to solid ice at 0℃ by a process called solidification.
- D to E, heat is released below the freezing point of ice thus supper cooling of the liquid i.e. **Supper cooling** is the fall of temperature of a substance below its freezing point with out change of state.

Related explanations

A small body cools faster compared to a big body of same material because the rate of fall off temperature of a body is inversely proportional to its mass. Since the mass of the body is proportional to its volume and the rate of heat loss is proportional to the surface area of the body, then the rate of temperature fall is proportional to the ratio of surface area to volume of a body. The ratio of surface area to volume is inversely proportional to the linear dimensions thus small bodies cool faster than big ones.

Metallic utensils being good conductors of heat, they absorb heat (from food) which would be carried away by the volatile liquid to the cooling fins thus delaying the refrigerating process. Such utensils are not recommended to be used in refrigerators.

Milk in a bottle wrapped in a wet cloth cools faster than that placed in a bucket exposed to a drought. This is because the wet cloth speeds up the rate of evaporation thus more cooling.

It advisable for a heavily perspiring person to stand in a shade other than drought because drought speeds up evaporation thus faster cooling which may lead to over cooling of the body and eventually this over cooling may lower the body's resistance to infections.

When taking a bath using cold water, the individual feels colder on a very shiny day than on a rainy day because on a shiny day, the body is at high temperatures such that on pouring cold water on the body, water absorbs some of the body's heat thus its cooling. Yet on a rainy day the body is at a relatively low temperature implying that less heat is absorbed from it when cold water is poured on it.

Two individuals; **A** (suffering from serious malaria) and **B** (normal) taking a bath of cold water at the same time of the day, **A** feels colder than **B** because the sick person's body is at relatively higher temperature than of a normal person. When cold water is poured on the sick person's body, much heat is absorbed from it compared to that absorbed from a normal person thus more coldness.

Two normal identical individuals; **A** (takes a bath of water at 35℃) and **B** (takes a bath of water at 25℃) after the bath, **A** experience more coldness than **B**. This is because Water at 35℃ raises the body's temperature more than that at 25℃. This means that after the bath, the individual who takes a bath of water at 35℃ looses more heat to the surrounding than what one who takes a bath of water at 25℃ would loose to it.

Water bottles are made of plastic other than glass and not fully filled because when water cools, it expands such that ice takes up a bigger volume. The un filled space is to cater for increase in volume on solidification and the bottle is made plastic to with stand breaking due to increase in volume.

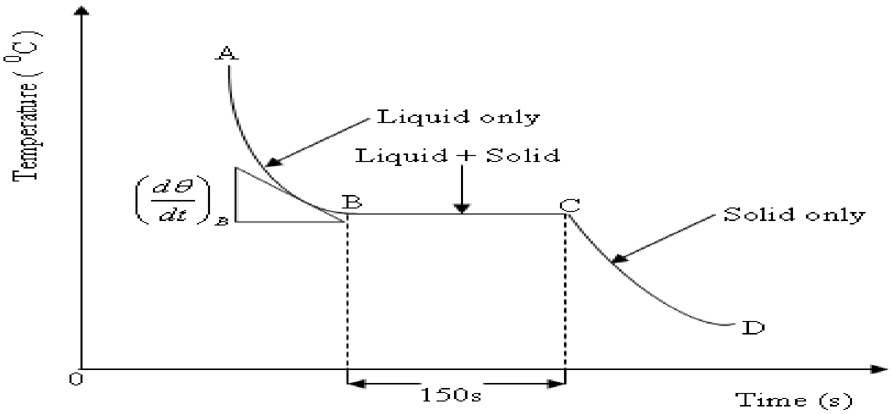
Example

A calorimeter of heat capacity $500JK^{-1}$ is filled with 2kg of a liquid and the cooling curve of the liquid is obtained. The rate of temperature fall just before solidification starts is $9.0Ks^{-1}$ and complete solidification takes 4.2minutes.

i) Sketch and explain the cooling curve obtained.

ii) If specific heat capacity of the liquid is $495Jkg^{-1}K^{-1}$ find the specific latent heat of fusion for the liquid.

Solutions:



Explanations:

- The cooling of liquid along AB is due to heat loss to the surroundings.
- Temperature along BC remains constant because the would be heat lost to the surrounding is compensated for by the latent heat of fusion released during conversion of liquid state to solid state.
- The fall of temperature along CD is due to the cooling of the solid as it looses heat to the surroundings.

(ii) In region BC

$$(Theamountofheatlostbythecalorimeter + thatlosttheliquid$$
$$= Theamountofheatthatsolidifiestheliquid)$$

Note that the rate of temperature fall $\left(\frac{\Delta\theta}{\Delta t}\right)_B$ is constant throughout BC

$C\left(\frac{\Delta\theta}{\Delta t}\right)\Delta t + m_l c_l\left(\frac{\Delta\theta}{\Delta t}\right)\Delta t = m_l L_f$

500x9x4.2x60 +2x495x9x4.2x60 =2L_f from which L_f = 1689660Jkg^{-1}.

Exercise

1) (a) (i) Explain why water bottles are plastic and not fully filled.

(ii) Explain why temperature of a liquid remains constant when it is boiling.

(iii) Use kinetic theory to explain why a liquid cools when it evaporates.

(b) (i) Explain why cooking with a pressure cooker is faster than ordinary cooking.

(ii) Explain what may happen when a person is to cook food from a very high altitude.

c) (i) Define boiling point and freezing point.

(ii) Describe an experiment to show that boiling only occurs when saturated vapor pressure is equal to external pressure.

(d) A calorimeter of heat capacity 700JK^{-1} is filled with 2kg of a liquid whose freezing point is 88℃ and initially at 150℃. The cooling patterns of the liquid are studied up to 35℃.

It is found out that the rate of temperature fall liquid just before solidification starts is $8.87s^{-1}$ and complete solidification takes 6.39minutes.

i) Sketch and explain the cooling curve obtained.

ii) If specific heat capacity of the liquid is $654Jkg^{-1}K^{-1}$ calculate the specific latent heat of fusion for the liquid.

2) (a) (i) Define ice point and steam point.

(ii) Describe how boiling point of alcohol may be determined.

(iii) Explain how pressure affects melting point of a substance.

(b) (i) Define evaporation and supper cooling.

(ii) State ways of increasing the rare of evaporation of a liquid.

(iii) Distinguish between evaporation and boiling.

c) (i) Define melting and cooling.

(ii) State factors which affect the body's rate of cooling.

(iii) Explain how **shape** and **size** of body's enclosure affects its rate cooling.

3) (a) (i) Define melting point and freezing point.

(ii) Describe a simple experiment for determining freezing point and melting point of Naphthalene.

(b) (i) State Newton's law of cooling.

(ii) Describe how the law stated in 3 b(i) above can be verified.

(iii) Use Newton's law of cooling to show that the body's rate of temperature fall is directly proportional to the excess temperature $(\theta_s-\theta_t)$ where; θ_s is temperature of the surrounding and θ_t is temperature of the body at time **t**.

c) Explain the following observations

(i) Two individuals "**A**" (suffering from malaria) and **"B"** (normal) taking a bath of cold water, A feels colder than B.

(ii) When taking a bath using cold water, the individual feels colder on a very shiny day than on a rainy day.

(iii) Milk in a bottle wrapped with a wet cloth cools faster than that placed in a bucket exposed to drought.

(iv) Metallic utensils are not recommended to be used in refrigerators.

(v) Small bodies cool faster than identical larger bodies placed in the same drought.

(vi) A bare cemented floor is felt colder than a carpeted floor.

4) (a) Equal volumes of water at the same temperature are poured in a basin and jug exposed to the same drought. Explain which water evaporates faster.

(b) Use kinetic theory to explain the following observations

(i) Ice melts faster when salt is sprinkled on it

(ii) Liquids expand more than equal volumes of solids when heated through the same temperature change.

c) 2.0kg of ice initially at -10℃ is heated until when it changes to steam at 100℃.

(i) Sketch and explain the graph showing temperature against time.

(ii) Calculate the internal energy required in each phase if (s.l.h.f of ice=3.36x105Jkg^{-1}, s.h.c of ice =2100Jkg^{-1}K^{-1} and s.*l*.h.v of water 2.26x106Jkg^{-1}).

Vapors

When a liquid in a closed container is heated some molecules leave its surface and occupy the space just above it.
These molecules constitute what we call **vapor** and the pressure they exert to the walls of the container as they collide with them selves and the walls of the container is called **vapor pressure**.
When these molecules bounce off from the walls of the container, they strike the liquid surface and re-enter the liquid until when a state of thermo dynamic equilibrium is attained i.e. (rate of evaporation =rate of condensation).
In this state vapor is said to be saturated exerting saturated vapor pressure and before this state, vapor is un saturated (with rate of condensation > rate of evaporation) exerting un saturated vapor pressure**.**
Therefore;
(i) Vapor is the gaseous state of a substance below its critical temperature i.e.

- *Critical temperature (T_c)* is the minimum temperature above which the gas can not be liquidized no matter how much pressure is applied.
- *Critical pressure (P_c)* is the minimum pressure needed to liquidize the gas at its critical temperature.
- *Critical volume (V_c)* is the volume occupied by a gas at its critical pressure and critical temperature.
- *Specific critical volume* is the volume occupied by 1kg mass of a gas at its critical pressure and critical temperature.
- *Specific critical pressure* is the minimum pressure needed to liquidize 1kg mass of a gas at its critical temperature.

(ii) Saturated vapor is one which is in thermal dynamic equilibrium with its own liquid i.e. whose rate of evaporation = rate of condensation.
(iii) Un saturated vapor is one which is not in thermal dynamic equilibrium with its own liquid i.e. whose rate of evaporation ≠ rate of condensation.
(iv) **Supper saturated vapor** is a vapour which contains more molecules than it should at a given temperature and is not in equilibrium with it's own liquid.
(v) Thermal dynamic equilibrium is the liquid's thermal state at which its rate of evaporation is equal to its rate of condensation.
(vi)Vapor pressure is one exerted on the walls of the container by the vapor molecules
(vii) Saturated vapor pressure (s.v.p) is one exerted by vapor which is in thermal dynamic equilibrium with its own liquid.

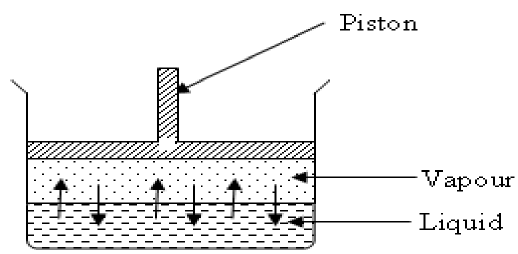
(viii) Un saturated vapor pressure is one exerted by vapor which is not in thermo dynamic equilibrium with its own liquid.
(ix) Supper saturated vapor pressure is one exerted by vapor whose rate of evaporation > its rate of condensation.
(x) Dew point is defined as temperature saturated atmospheric air.
(xi) Gas is a gaseous state above critical temperature.
NB: A cloudy film forms on screens of cars being driven in rain because of the condensation of the excess water vapor in atmospheric moist air as a result of exceeding its dew point.

Comparison of vapor pressure

Saturated vapor.	**Un saturated vapor**.
It does not obey ideal gas laws.	It obeys ideal gas laws.
It is achieved at thermal dynamic equilibrium.	Its rate of evaporation ≠its rate of condensation.
Its pressure remains constant at particular temperature.	Its pressure increases with increase in temperature.

Kinetic theory explanation for the occurrence of saturated vapor pressure (s.v.p)

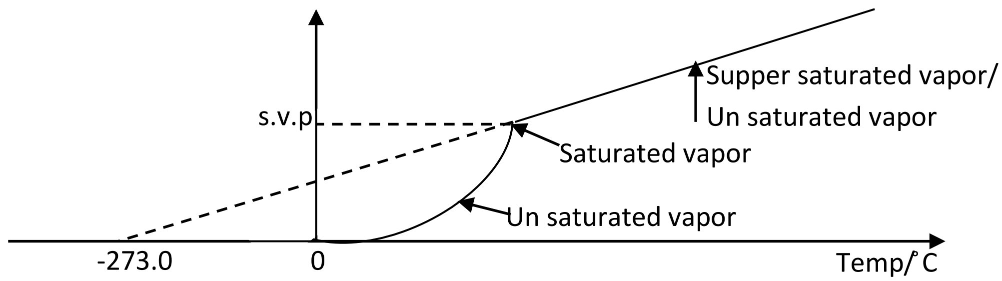
- When a liquid is heated in a closed container, its molecules gain kinetic energy and evaporate to form vapor just above the liquid surface.
- These vapor molecules are free and randomly collide with each other.
- The collision make less energetic molecules to be pushed back into the liquid as the energetic one remaining leaving the liquid surface until when the rate of evaporation is equal to the rate of condensation.
- At this state the vapor is said to be saturated and exert saturated vapor pressure to the walls of the container.



Vapor pressure and temperature

- Initially vapor pressure increase slowly with increase in temperature exponentially because fewer molecules are energetic to leave the liquid surface but as the liquid's boiling point is approached, vapor pressure rapidly increases i.e. un saturated vapor pressure.
- At boiling point vapor pressure remains constant (saturated vapor pressure) since vapor is saturated.

- Heating the liquid beyond its boiling point results into supper saturated vapor whose rate of evaporation is greater than its rate of condensation and vapor pressure (un saturated



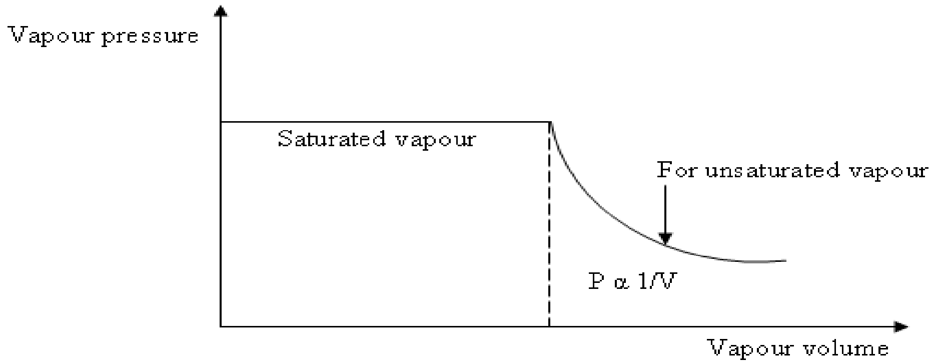
- Vapor pressure increases linearly with increase in temperature due to increased multiple collisions of vapor molecules with the walls of the container.

Vapor pressure and vapor volume

- A reduction in vapor volume of saturated vapor only leads to an equal reduction in the number of vapor molecules above the liquid surface. Since s.v.p is achieved at constant temperature (boiling point), the vapor remains in thermal dynamic equilibrium. This means that the force per squared meter exerted on the walls of the container remains constant due to an equal reduction in the surface of the walls for colliding molecules.
  Question: Explain how increasing vapor volume of saturated vapor would affect the pressure it exerts.
- Since un saturated vapor is in position to accommodate more vapor molecules, reducing vapor volume implies compiling a larger number of vapor molecules in a smaller surface above the liquid. This results into increased number of collisions of such vapor molecules with surface of the walls of the container thus increased pressure exerted.

Question: Explain how increasing vapor volume of un saturated vapor would affect the pressure it exerts.

This therefore means that saturated vapor pressure does not obey Boyle's law of perfect gases yet un saturated vapor pressure does as shown below.



NB:

Saturated vapors do not obey ideal gas laws because its mass changes due to condensation or evaporation as conditions change yet gas laws only apply to a constant mass of a gas.

It should be noted that saturated vapor occurs for a very short time and constant temperature (boiling point).

Use kinetic theory of matter to explain the observation that saturated vapour pressure of a liquid increases with temperature.

Increase in temperature increases the kinetic energy of the liqud molecules and the rate of evaporation increases. Therefore the rate of condensation also increasesthus pressure of the vapour rises. As the rate at which molecules bombard the liquid increases. The dynamic equilibrium is restored at a higher saturated vapour pressure.

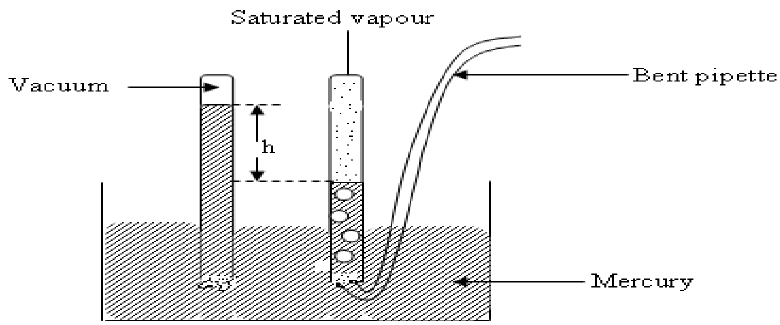
Determining saturated vapor pressure

Bent-pipette method

This method is used to determine the **s.v.p** of a volatile liquid at a particular temperature.

Procedure

The volatile liquid whose **s.v.p** is to be determined is introduced with the help of a bent pipette in the vacuum space above the mercury level in one of the barometric tubes as shown below.



Some of the volatile liquid evaporates to fill the space above the mercury and its vapor exerts a pressure on the mercury causing it to drop.
When mercury has stopped dropping, the vapor is said to be in dynamic equilibrium, thus saturated vapor.
The pressure $h\rho g$ is the **s.v.p** of the volatile liquid and **ρ** is its density.

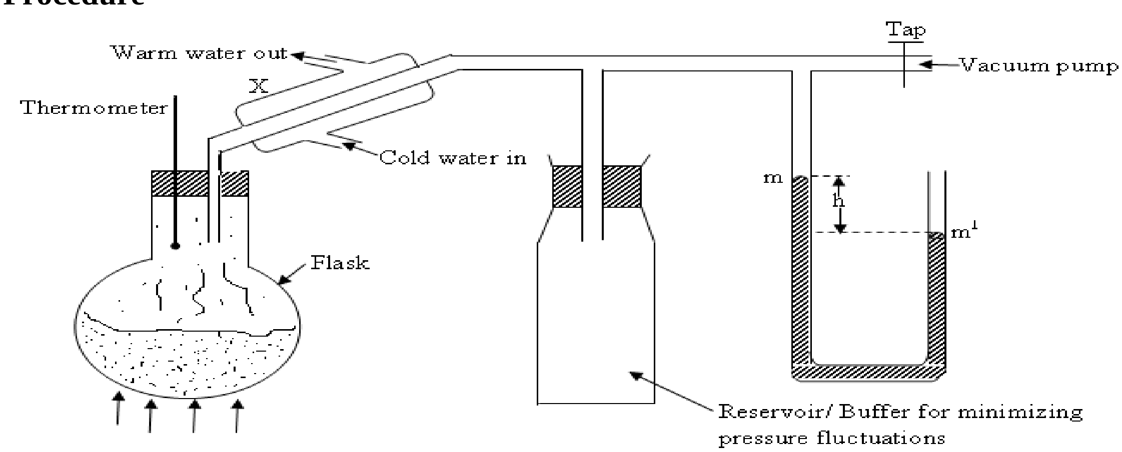
Merits of mercury for this experiment

Mercury is very dense compared to many liquids
Mercury is opaque thus easily seen and read.

Dynamic method

This is used to determine s.v.p at different temperatures.

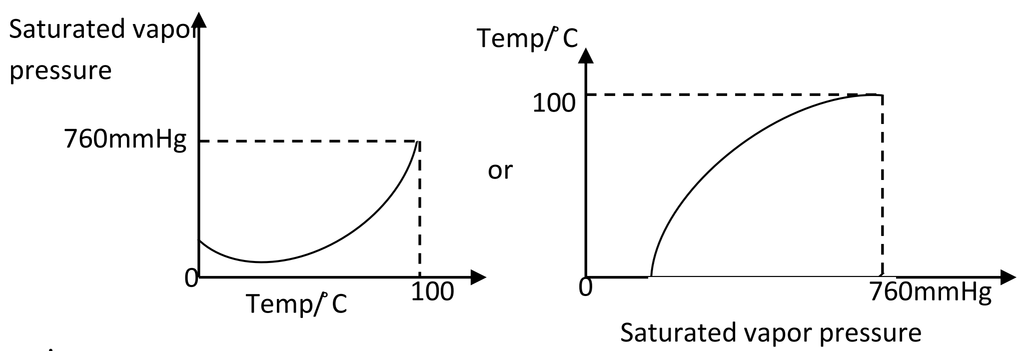
Procedure



Air is pumped in and out from the reserviuor by using a pump when the tap is opened. The tap is closed and the liquid is heated until its boling point. The temperature θ of the vapour is determined. The difference h, in the mercury levels is noted from the manometer. The atmospheric pressure is measured using a manometer. The pressure of the vapour $P = H + h \ or \ H - h$ and is the saturated vapour pressure.

For variation of saturated vapour pressure with temperature,

The procedure above is repeated for different pressures. A graph of P against temperature is plotted and a straight line graph is obtained which shows that saturated vapour pressure increases with temperature.



Question

1. Describe an experiment to determine saturated vapour pressure of a liquid.
2. Describe an experiment to investigate the variation of saturated vapour pressures with temperatures.

NB: s.v.p increases with increase in temperature because;

- Increasing temperature increases the average kinetic energy of liquid molecules thus increasing their rate of escape from the surface.
- This result into increased number o f vapor molecules above the liquid surface which leads to multiple collisions of such molecules with the walls of the container thus increased vapor pressure (s.v.p).

Solving problems which involve vapors

We need to identify whether the problem involves a mixture of a gas and an unsaturated vapor or saturated vapor i.e.

For the mixture of a gas and unsaturated vapor, apply the equation of state on the mixture since they both obey Boyle's law.

For the mixture of a gas and **saturated vapor** for which the gas obeys Boyle's law but the vapor does not obey Boyle's law, we apply Dalton's law of partial pressure to the mixture to get the air pressure (P_{air}) and saturated vapor pressure (P_{vapor}) i.e. $P_{total} = P_{air} + P_{vapor}$ and then apply equation of state on the gas only.

Examples

1) A volume of $4.0\times10^{-3}m^3$ of air is saturated with water vapor at 100℃. This air is cooled to 20℃ at a pressure of 1.33×10^5Pa. Calculate the volume of air after cooling if the s.v.p of water at 20°C is 2.3×10^3Pa and pressure of dry atmospheric air at 100℃ is 1.01×10^5Pa.

Solutions:

At 20℃, $P_T=1.33\times10^5$Pa, $P_s=2.3\times10^3$Pa, $P_{air}=P_T-P_s=1.307\times10^5$Pa.

Now we apply the equation of state on air alone i.e.

$$\frac{1.01\times10^5\times4.0\times10^{-3}}{(100+273)}=\frac{1.307\times10^5\times V_{20}}{(20+273)}$$

$$V_{20}=2.428\times10^{-3}m^3$$

2) A closed vessel contains air saturated with water vapor at 60℃. The total pressure in the vessel is 1000mmHg. Calculate the new air pressure if its temperature is reduced to 27℃. Take s.v.p of water vapor at 60℃ to be 314mmHg and 112mmHg at 27℃.

Solutions:

Total pressure (P_T) =1000mmHg. At 60℃, s.v.p=314mmHg

$$\Rightarrow P_a=P_T-P_s=1000-314=686mmHg$$

Now we apply the state equation to air alone i.e.

$$\frac{686}{(273+60)}=\frac{P_{27}}{(273+27)}$$

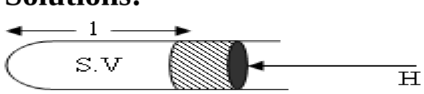
$$P_{27}=\frac{686*300}{333}=618mmHg$$

$(P_T)_{27}=(P_{air})_{27}+(s.v.p)_{27}$

$= 618 + 112 = 730$mmHg.

3) A column of air is sealed in a horizontal uniform capillary tube by a water index which is sufficient to saturate the air. At 20℃, the length of the air column is 15.6cm at atmospheric pressure of 762.5mmHg. When the air in the capillary tube is heated to 50℃, the length of air column becomes 19.1cm at the same atmospheric pressure. If the saturated vapor pressure of water at 20℃ is 17.5mmHg, what is its value at 50℃?

Solutions:



At 20℃, P_T=762.5mmHg, s.v.p (P_s) =17.5mmHg, P_{air}= (762.5-17.50) =745mmHg

Now applying state equation to air alone

Initial conditions: Temp = 20℃, Volume = 15.6cm^3 and Pressure = 745mmHg

Final conditions: Temp =50℃, Volume = 19.1cm^3 and Pressure = P_a^1

$$\frac{74.5*16.6}{293}=\frac{P_a^1*19.1}{323}$$

$\Rightarrow P_a^1 = 670.8mmHg$

Now applying Dalton's law of partial pressure on the mixture at 50℃, we have

$P_s^1 = P_T - P_a^1 = 762.5 - 670.8 = 19.7mmHg$

4) A uniform capillary tube sealed at one end contains air enclosed by a thin film of water which keeps it saturated with vapor. The length of air column is 15.6cm at 20℃ and 25cm at 70℃. Assuming s.v.p of water to be 17mmHg at 20℃, calculate the s.v.p of water at 70℃ (Hint: take total pressure at 20℃ to be 88cmHg and 104cmHg at 70℃).

Solution: At 20℃, P_a= (88-1.7) =86.3cmHg.

Now applying state equation to air only,

$$\frac{86.3*15.6}{293}=\frac{P_a^1*25}{343}$$

$\Rightarrow P_a^1 = 63.04cmHg$

Now applying Dalton's law to the mixture at 70℃;

5) Moist air at 50°C and pressure of 760mmHg is contained in a sealed vessel. When the vessel is cooled, saturation starts at 20℃. What will be the total pressure at 10℃? (s.v.p of water at 20℃ and 10℃ are 17.5mmHg and 9.0mmHg respectively).

Solutions:

- Since the vessel is sealed, then V is constant.
- Moist air is a mixture of air and water vapor. This means that since saturation starts at 20℃, the vessel contains air and unsaturated vapor between 50℃ to 20℃ and for 20℃ to 10℃ it contains air and saturated vapor.
- Since such mixture obeys Boyle's law between 50℃ to 20℃ we can apply the equation of state to it during this temperature interval.

For 50℃ to 20℃ $\frac{760}{323}=\frac{P_T}{293} \Rightarrow (P_T)_{20} = 689.4mmHg$

For 20℃ to 10℃, its only air which obeys Boyle's law

Applying Dalton's law at 20℃: $P_{air} = 689.4 - 17.5 = 671.9mmHg$

Now we apply state equation to air alone i.e.

$\frac{671.9V}{293}=\frac{P'_aV}{283}$ From which P_{air} at 10℃ =649.0mmHg

Therefore when we apply Dalton's law of partial pressures we get;

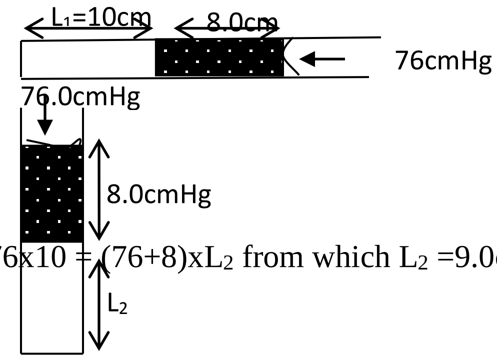
$(P_T)_{10}$ =649.0+9.0 =658.0mmHg

6)A column of air 10cm long is trapped in a horizontal uniform capillary tube by a mercury thread 8cm long. One end of the tube is open to the atmosphere and the other is closed. Find volume of air when the tube is held vertically with the open end exposed to the atmosphere.

Solutions:

For horizontal posture

For vertical posture



For $P_1L_1 = P_2L_2 \Rightarrow 76x10 = (76+8)xL_2$ from which L_2 =9.0cm.

Exercise

1) (a) (i) Definesaturated vapors and supper saturated vapors.

(ii) Describe an experiment to investigate temperature dependence of saturated vapor pressure of water.

(b) Explain the following observations

(i) Saturated vapor pressure does not obey Boyles's law 0f perfect gases.

(ii) Vapor pressure increases with increase in temperature.

c) (i) Explain how increasing vapor volume of saturated vapors would affect the pressure they exert.

(ii) Explain how decreasing vapor volume of un saturated vapors would affect the pressure they exert.

2) (a) Use kinetic theory to explain;

(i) The occurrence of saturated vapor pressure

(ii) The increase in vapor pressure when temperature of a liquid is increased.

(b) Write short notes on the following terms;

(i) Critical volume

(ii) Critical pressure

(iii) Critical temperature

(iv) Specific critical volume

(i) Specific critical pressure

c) (i) Define vapor and dew point.

(ii) A sealed vessel contains a mixture of air and saturated vapor with total pressure of $1.0x10^5$pa and $1.3x10^5$pa at 27℃ and 60℃ respectively. Determine saturated vapor pressure of water at 60℃ if s.v.p at 20℃ is 23000pa.

3) (a) (i) Distinguish between saturated vapors and un saturated vapors.

(ii) Describe an experiment to determine saturated vapor pressure of water.

(b) A volume of $0.004m^3$ of air at pressure of $1.01x10^5$pa is saturated with water vapor at 100℃. This moist air is cooled to 20℃ at total pressure of $1,33x10^5$pa. Calculate the new volume of air at 20℃ if s.v.p at 20℃ is $2.3x10^3$pa.

c) Use sketch graphs only to show how vapor pressure varies with;

(i) Vapor volume

(ii) Temperature

(d) S.v.p of ether at 0℃ is 185mmHg and 440mmHg at 20℃. Constant volume-gas thermometer whose bulb contain both dry air and sufficient ether to saturate air measures 100mmHg at 20℃. Determine the value it measures at 0℃.

4) (a) (i) State with reason two merits of using mercury in a barometer.

(ii) Describe a simple experiment to investigate the effect of pressure on boiling point of water (Hint describe the dynamic method).

(b) Describe an experiment for determining saturated vapor pressure of volatile liquids.

c) Air confined in a container at 27℃ and total pressure of $1.23x10^5$pa is saturated with water vapor. This air remains saturated at 77℃ and total pressure of $1.64x10^5$pa. Calculate the s.v.p of water at 77℃ if its value at 27℃ is 3600pa.

(d) A column of air is sealed in a horizontal uniform capillary tube by water index which is just sufficient to saturate air. At 20℃ the length of air column is 15.6cm at total pressure of 762.5mmHg. When air in the capillary tube is heated to 50℃, the air column becomes 19.3cm at the same total pressure. Calculate the value of s.v.p at 50℃ given that its value at 20℃ is 17.5mmHg.

(e) Moist air at 56.5℃ and pressure of 760mmHg is contained in a sealed vessel. When the vessel is cooled, saturation starts at 30.7℃. saturated vapor pressure of water at 30.7℃ and 12.5℃ are 67.5mmHg and 19.0mmHg respectively. Calculate the total pressure of air at 12.5℃.

LAWS OF PERFECT GASES

Laws of ideal gases include;

- Boyle's law which assumes constant temperature of the gas.
- Pressure law which assumes constant volume of the gas.
- Charles's law/Gay-lussac's law which assumes constant pressure exerted by the gas.

NB$_1$:

✓ The free volume "**V"** of the container the gas can occupy, pressure "**P"** exerted by the gas to the container and thermodynamic temperature **"T"** of the gas are called the **variables**.

✓ In verification of all the above laws, the mass of the gas under investigation is fixed/kept constant such that any change noticed is only due to changes in the variables being investigated.

Kinetic theory of ideal gases

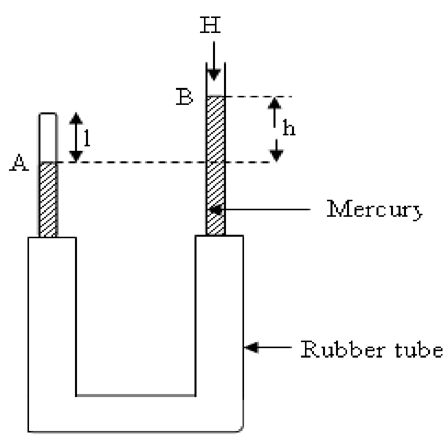
This theory looks at perfect gases as those with;

- The volume of the molecule is a negligibly small fraction of the volume occupied by the gas.
- Total number of molecules is large.
- Fast moving molecules and that their continual bombardments with the walls of the container they occupy constitute the pressure they exert to it.
- Molecules continuously and perfectly colliding thus no loss of their kinetic energy due to their collisions.
- Negligible intermolecular forces of attraction during collision.
- Molecules whose duration of collision is negligible compared to time between their collisions.
- Molecules which travel in a straight line.

Boyle's law of perfect gases

This law states that pressure of a fixed mass of an ideal gas is inversely proportional to its volume at constant temperature i.e. PV = constant.

Verification.

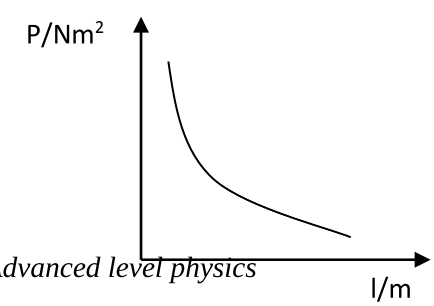


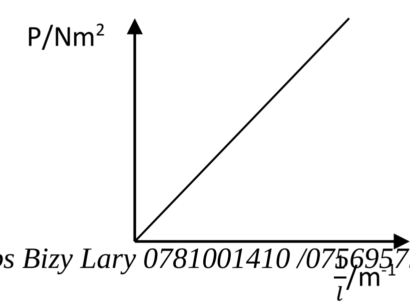
Dry ideal gas is trapped in a closed limb of a U tube by mercury as shown above.

Pressure exerted by this gas on mercury in the closed limb is varied by exerting a compressive force at the open limb of the tube. This pressure is given by P =H+h for "B" above "A" or P =H-h for "B" below "A".

Different values of **h** and their corresponding values of ***l*** are measured using a meter rule and recorded in a suitable table including values of P.

A plot of P against *l* is and that of P against $\frac{1}{l}$ is





Any of the above plots verifies Boyle's law of ideal gases.

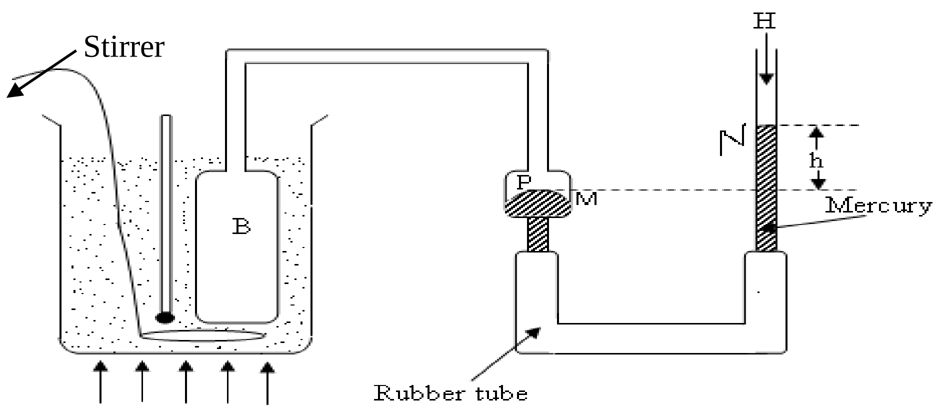
Kinetic theory explanation for Boyle's law

When the open limb of the "U" tube is compressed, mercury in the closed limb rises up reducing the available free volume of the gas above it. This slows down the gas molecules as they are brought closer to each other making more collisions than before. The increased number of collisions results into increased pressure the gas exerts on mercury in the closed limb and the walls of the tube thus Boyle's law**.**

Pressure law of perfect gases

This law states that pressure of a fixed mass of an ideal gas is directly proportional to its absolute temperature at constant volume of the gas i.e. $\frac{P}{T} = constant$

Verification.



Dry ideal gas of fixed mass trapped in bulb B is heated to increase its temperature by heating the liquid which submerges B.

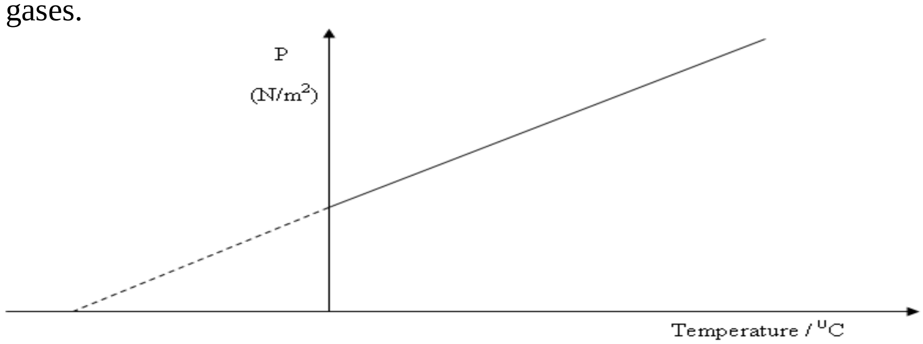
The gas molecules gain kinetic energy and move randomly exerting pressure on mercury in the closed limb (displacing it down wards below mark "M").

Mercury level in the open end of the tube is adjusted by compressing the tube such that mercury in the closed end is brought back to mark **M** to ensure constant volume of the gas above mercury in the closed limb.

The measured values of 'h" when mercury in the closed limb is at mark M and the corresponding read values of temperature from the thermometer are recorded including values of P=H+h for "M" below "N" or

P =H-h for "M" above "N".

A plot of P against temperature is linear as shown below thus verifying pressure law of ideal gases.



Kinetic theory explanation for pressure law of perfect gases

When temperature of the gas is increased, the average kinetic energy of gas molecules increase and molecules move faster.

But since its volume is constant, the increase in speed results into rapid collisions of the gas molecules with the walls of the container thus increase in pressure exerted to the walls.

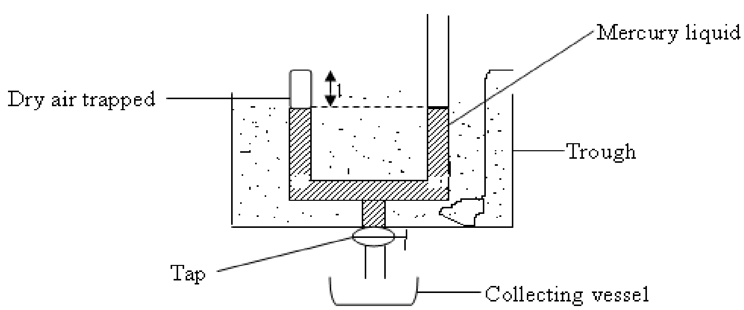
Question: Explain why pressure of a fixed mass of a gas increases when volume is reduced at constant temperature.

At constant temperature, the verage kinetic energy of the molecules is constant. Reducing the volume of a gas pushes the molecules together so that the number of collisions of the molecules hitting the wall increases, hence pressure increases.

Charles's law/Gay-lussac's law of perfect gases

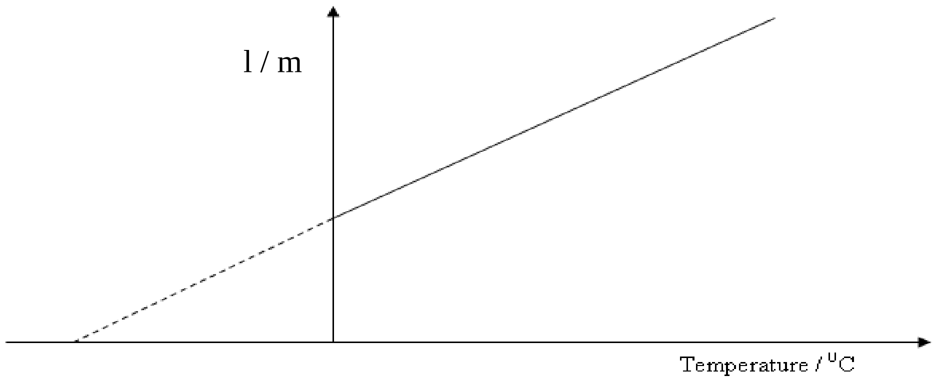
This law states that volume of a fixed mass of an ideal gas is directly proportional to its thermodynamic temperature at constant pressure of the gas i.e. $\frac{V}{T} = constant$

Verification



Dry ideal gas is trapped in the closed limb and its temperature is varied (increased) by passing steam into the water bath Due to increase in temperature, the gas exerts pressure on mercury in the closed limb such that mercury is displace upwards in the open limb.

The liquid is stirred and at steady state, mercury in the limbs is leveled by releasing some out through the tap to ensure constant pressure (atmospheric pressure).
The value of ***l*** is measured by a meter rule and recorded including the thermometer reading.
Several values of *l* are obtained at various corresponding temperature values and a plot of **l** against **temperature** is linear as shown below thus verifying Gay-lussac's law of ideal gases.



Kinetic theory explanation for Charles's law of perfect gases
When a gas is heated, average kinetic energy of the gas molecules increases. This results into increased number of collisions such that for pressure exerted by the gas on mercury and walls of the tube to remain constant distance between collisions must be made longer than before thus increased volume due to increased temperature.
Likewise, decrease in temperature leads to a decrease in the average kinetic energy of the gas molecules. This result into fewer collisions such that to maintain a constant pressure exerted by the gas, distance between collisions must be reduced thus reduction in volume of a gas as stated by Charles's law of ideal gases.

Thermal coefficients of ideal gases

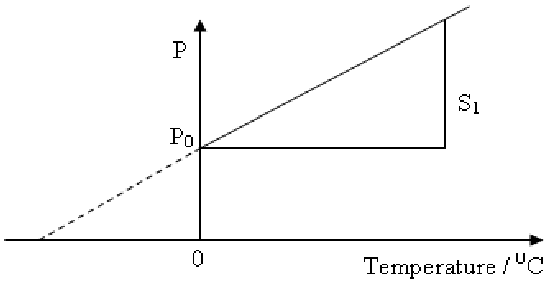
They are:
Coefficient of thermal expansion of a gas at constant volume (α_v) is the fractional increase in the pressure exerted by an ideal gas for every 1K rise in its temperature**.**
Therefore $\alpha_v = \frac{P_\theta - P_0}{P_0\theta}$ since $P_\theta = P_0(1+ \alpha_v\theta)$.
Coefficient of thermal contraction of a gas at constant volume (α_v) is the fractional decrease in the pressure exerted by an ideal gas for every 1K rise in its temperature**.**
Therefore $\alpha_v = \frac{P_\theta - P_0}{P_0\theta}$ since $P_\theta = P_0(1+ \alpha_v\theta)$.
Coefficient of thermal expansion of a gas at constant pressure (α_p) is the fractional increase in the volume of an ideal gas for every 1K rise in its temperature.
Therefore $\alpha_p = \frac{V_\theta - V_0}{V_0\theta}$ since $V_\theta = V_0(1+ \alpha_p\theta)$.

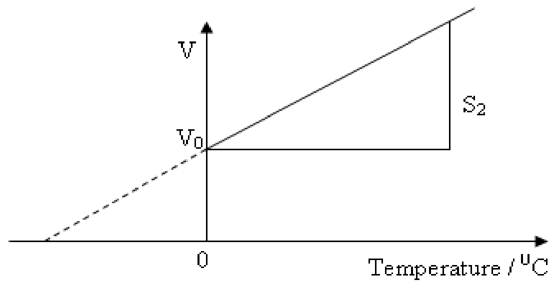
Coefficient of thermal contraction of a gas at constant pressure (α_p) is the fractional decrease in the volume of an ideal gas for every 1K rise in its temperature.

Therefore $\alpha_p = \frac{V_\theta - V_0}{V_0 \theta}$ since $V_\theta = V_0(1 + \alpha_p\theta)$.

NB:

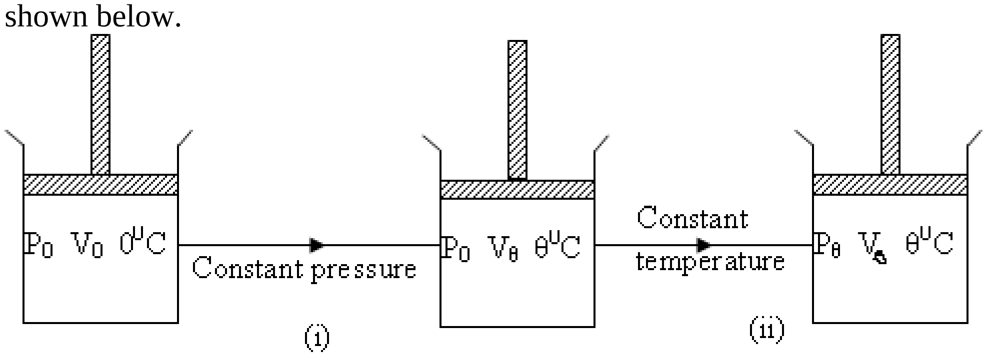
- α_v can be determined experimentally by verifying pressure law such that the slope of the graph obtained is given as $P_0\alpha_v$ = Slope (S_1).
- α_p can be determined experimentally by verifying Charles's law such that the slope of the graph obtained is given as $V_0\alpha_p$ = Slope (S_2).





Proof that $\alpha_v = \alpha_p$ for ideal gases

We consider an ideal gas initially at 0°C, pressure P_o and volume V_o to undergo changes as shown below.



Since in (i) temperature raises, the gas expands from V_o to V_θ such that

$V_\theta = V_o(1 + \alpha_p\theta)$ …………(i)

consider step (ii) for constant temperature implying that increasing pressure from P_o to P_θ compresses the volume of the gas from V_θ back to V_o such that by Boyle's law $P_oV_\theta = P_\theta V_o$ ……(ii)

Now substituting for $P_\theta = P_o(1 + \alpha_v\theta)$ and $V_\theta = V_o(1 + \alpha_p\theta)$ into (ii) yields

$P_oV_o(1 + \alpha_p\theta) = P_oV_o(1 + \alpha_v\theta)$

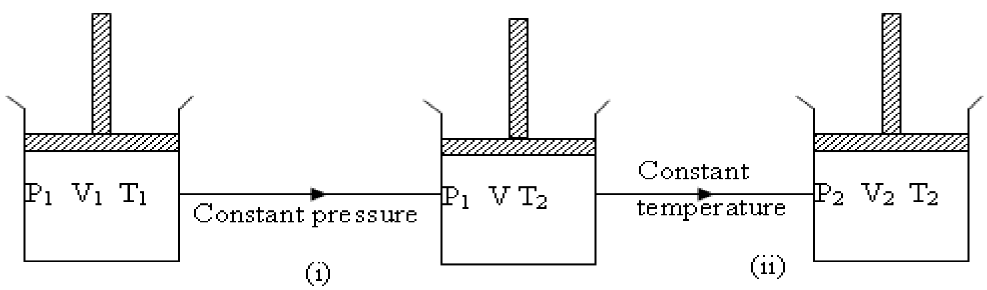
$\Rightarrow \alpha_p = \alpha_v$

N.B: Any discrepancy in **α_v** and**α_p** is a measure of the gas's departure from ideal state.
Example

1. The bulb of a costant volume gas thermometercontains air and sufficient water to saturate the space up to 100^0C. At 85^0C, the pressure is 132cmHg, and at 45^0C, the pressure is 89cmHg. If the saturated vapour pressure of water is 6.4cmHg at 450C and 37.2cmHg at 85^0C. Calculate the expansivity of air at constant volume. (4.43×10^{-3})

Ideal gas equation/ state equation

The relationship between pressure,P, volume V and absolute temperature Tof a fixed mass of gas which approximates to ideal state i.e. $|\alpha_v-\alpha_p|\approx0$ is called its equation of state.
Derivation.
We consider a gas initially at T_1, P_1, and V_1 to go through changes shown below.



During step (i) Charles's law is obeyed i.e.$\frac{V_1}{T_1}=\frac{V}{T_2}$ from which V=$\frac{V_1T_2}{T_1}$ …..(i).
During step (ii) Boyle's law is obeyed i.e.$P_1V=P_2V_2$ from which V =$\frac{P_2V_2}{P_1}$….(ii)
Equations (i) and (ii) are equal i.e. $\frac{P_2V_2}{P_1}=\frac{V_1T_2}{T_1}$

$\Rightarrow\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}=constant$ …. (iii)

N.B$_1$:
It should be noted that since the gas is not perfectly ideal, it does not necessary mean that when pressure is increased from P_1 to P_2 volume is compressed from V_2 to V_1 as the case for pure ideal gases.

(a)For 1 mole of an ideal gas, equation (iii) gives the universal/ molar gas constant (**R**) i.e. $\frac{PV}{T}=R$ Where; **V** is the free volume of a container a gas can occupy, **P** is the pressure exerted on the walls of the container by the gas and **T** is the thermal dynamic temperature of the gas.

Molar gas constant (R) is the haet required to raise one mole of a gas by 1K

(b)At s.t.p (standard temperature and pressure) for which;

- Thermodynamic temperature, T=273K
- Atmospheric pressure P=hρg=760×10^{-3}×9.81×13600=101396.16Pa
- Molar volume V=22.4 liters =22.4dm^3i.e.

Since 1litre=1dm^3 =$10^{-3}m^3$ then;

22.4litres=22.4dm^3 =$\frac{10^{-3}x22.4}{1}$$m^3$.

Therefore, at s.t.p $R = \frac{101396.16 * 0.0224}{273} \Rightarrow R = 8.314 Jmole^{-1}K^{-1}$

(c) For a gas of **n** moles at s.t.p $\frac{PV}{T} = nR$, for which;

n =$\frac{N}{N_A}$=$\frac{Number\ of\ atoms}{Avogadro'sconstant}$ =$\frac{Known\ mass\ of\ the\ gas}{Molar\ mass\ of\ the\ gas}$.

Examples

1**)** What is the temperature of 19m^3 of an ideal gas at 600mmHg if the same gas occupied 12m^3 at 760mmHg at 27^oC?

Solutions:

Using the state equation

$$\frac{600*19}{T_1} = \frac{760*12}{300} \Rightarrow T_1 = 375K$$

2) A gas has a volume of 60cm^3 at 20^oC and 900mmHg. What would be its volume at s.t.p?

Solutions:

$$\frac{60*900}{293} = \frac{760*V_2}{273} \Rightarrow V_2 = 66.2cm^3$$

3) A vessel containing 400m^3 of a gas at a pressure of 8.7×10^{-2}Pa and temperature 20^oC is compressed isobarically to half its initial volume. Find the new temperature.

Solutions:

Isobaric process imply constant pressure

$$\frac{400}{293} = \frac{200}{T_2} \Rightarrow T_2 = 146.5K$$

4) A cylinder contains 2.4×$10^{-3}m^3$ of Hydrogen gas at 17^oC and 2.32×10^6Pa. The relative molecular mass of hydrogen is 2 and universal gas constant R=8.314$Jmol^{-1}K^{-1}$ and N_A=6.02×$10^{23}mol^{-1}$. Find

i) The number of moles of hydrogen in the cylinder
ii) The number of hydrogen atoms present in the cylinder
iii) The mass of hydrogen in the cylinder
iv) The density of hydrogen at that temperature

Solutions:

(i) Using $\frac{PV}{T} = nR$

$$n = \frac{2.32*10^{6} * 2.4*10^{-3}}{290*8.314} = 2.3 moles$$

(ii) Number of atoms $N = nN_A$, since $n = \frac{N}{N_A}$

$$N = 2.3*6.02*10^{23} = 1.38*10^{24} atoms$$

(iii) From $n = \frac{m}{M}$, $m = nM = 2.3*2 = 4.6g$

(iv) Density $= \frac{m}{V} = \frac{4.6*10^{-3}}{2.4*10^{-3}} = 1.916 kgm^{-3}$

5) A cylinder contains $0.25m^3$ of a gas at a pressure of 2.0×10^6Pa and a temperature of 17^oC. Find;

i) The number of moles of the gas in the cylinder

ii) The pressure of the gas if the temperature of the gas is raised to 37^oC at a constant volume of the gas.

Solutions:

i) $n = \frac{PV}{RT} = \frac{2.0*10^{6} * 0.25}{290*8.314} = 207 moles$

ii) Using $\frac{PV}{T} = constant$

$$\frac{2.0*10^{6} * 0.25}{290} = \frac{P_2 * 0.25}{310} \Rightarrow P_2 = 2.14*10^{6} Pa$$

6) The density of Oxygen at s.t.p is $1.43kg/m^3$. A 20litre cylinder is filled with Oxygen at pressure of 25 atmospheres at temperature of 27^oC. What is the mass of Oxygen in the cylinder?

Solutions:

From $PV = nRT = \frac{m}{M}RT = mR^1T$, where $R^1 = \frac{R}{M}$

Or $R^1 = \frac{PV}{mT} = \frac{P}{\rho T} = \frac{1.01*10^{5}}{1.43*273} = 259 Jmol^{-1}kg^{-1}K^{-1}$ at s.t.p

From $PV = mR^1T$. At 25 atmospheric pressure and 27^oC we have

$$m = \frac{25*1.01*10^{5} * 20*10^{-3}}{300*259} = 0.65kg$$

7) The volume of air in a tyre is 24.6 litres when the pressure is 2.0×10^5Pa at constant temperature.

i) What volume will this air occupy at a pressure of 1.0×10^5Pa?

ii) How much air will escape from the tyre when the valve is removed?

Solutions:

i) Using Boyle's law, PV=constant

$24.6*2.0*10^5 = 1.0*10^5 *V_2$

$\therefore V_2 = 12.3 litres$

ii) When the valve is removed, volume reduces from 24.6 litres to 12.3 litres at atmospheric pressure i.e.

New volume of the gas V = 24.6 -12.3 =12.3litres.

8. A balloon of weight 0.8N is filled with helium at 0℃ and 1 atmospheric pressure. The balloon contains 20 moles of helium. Find

i) The volume of the air in the balloon.

ii) The extra weight that the balloon can lift at the earth's surface. (molar mass of helium and air are 4.0g and 29.0g respectively)

Solutions:

i) Using $PV = nRT$

$$V = \frac{20*8.314*273}{1.01*10^5 *1} = 0.44m^3$$

ii) Weight of the balloon = 0.8N

Total weight the balloon can lift = weight of air displaced (by the law of floatation).

Mass of air displaced = Molar mass × Moles $= 29.0*10^{-3}*20$

Weight of air displaced = $29.0*10^{-3} *20*9.8 = 5.7N$

$\Rightarrow$ Total weight the balloon can lift is 5.7N.

Weight lifted by the balloon is = (its actual weight + weight of helium gas).i.e.

Mass of helium gas $= 4*10^{-3} *20$

Weight of helium gas $= 4*10^{-3} *20*9.8 = 0.78N$

$\therefore$ The extra weight = Total weight – (Weight of helium + Weight of balloon)

$= 5.7 - (0.8 + 0.78) = 4.1N$

VAN DER WAALS' EQUATION OF STATE

Van der Waals modified the ideal gas equation PV = RT by taking into account (modifying) two of these assumptions which may not be valid. He urged that:

- ***The volume of the molecules may not be negliginble compared to the volume V occupied by the gas.*** This accounts for the factor b.
- ***The attractive forces between the molecules are not be negigligible*** and this leads to the pressure defect and accounts for the term $\frac{a}{V^2}$.

Van der Waals said that the pressure P in the equation PV = RT is less than the true pressureby an amount p because of attractive forcesbetween molecules. According to him, this pressure defect p is proportional to the product of (a) number of molecules striking a unit areaof the wall per second, and (b) number of molecules per unit volume behind them exerting attractive forces. For a given volume, both these numbers are proportional to the density ρ of the gas so that the pressure defect p is proportional to ρ^2. For a given mass, $\frac{1}{V}$. hence $p\ \alpha \frac{1}{V^2}$, $p = \frac{a}{V^2}$, where a is a constant depending on the nature of the gas. Thus the true pressure of the gas is P + p = P + $\frac{a}{V^2}$.

He further suggested that the volume V is not the true volume of the gas because the molecules themselves opccupy a finite volume. The true volume of the gas according to him is (V – b)where b (co-volume) is a factor depending on the actual volume of the molecules themselves. Thus *van Der Waals equation for real gases is* $(P + \frac{a}{V^2})(V - b) = RT$.

NB

At high pressures when when the molecules are too many and too close together, the correction factors a and b both become important. But at low pressures, when they are not too many and too close together, a gas behaves like an ideal gas and obeys the equation PV = RT.

Exercise

1) (a) (i) State Boyle's Law

(ii) Use kinetic theory of ideal gases to account for Charles's Law.

(iii) Describe an experiment to verify pressure law of ideal gases.

(b) A cylinder contains 2.4×10^{-3} m^3 of Hydrogen at 20℃ and 2.0×10^6 pa. The relative molecular mass of Hydrogen is 2.0g and (R = 8.314 Jkg^{-1}mol^{-1}k^{-1}, NA = 6.02×10^{23} mol^{-1}). Find;

(i) The number of hydrogen molecules present in the cylinder.

(ii) The number of hydrogen moles present in the cylinder.

(iii)The mass of hydrogen in the cylinder.

(iv)The density of hydrogen at this temperature.

(c) Explain how pressure of the surrounding affects the rate of cooling of the body.

2) (a) (i) Define ideal gas equation.

(ii) State Charles's Law of ideal gases.

(iii) Use kinetic theory to account for pressure law of ideal gases.

(b) Describe an experiment to verify Boyle's law of ideal gases.

c) A cylinder contains 0.25m^3 of an ideal gas at a pressure of 2.0×10^6pa at 25°C. Find;

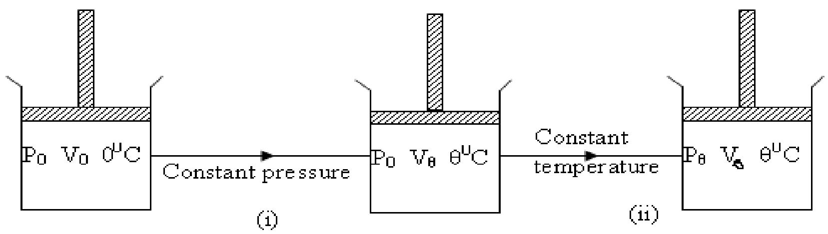
(i) number of moles of a gas in the cylinder.

(ii) Pressure of the gas if the temperature of the gas is raised to 37°C at constant volume.

(d) Explain how the thermodynamic scale of temperature is established.

3) (a) (i) Define coefficient of expansion of ideal gases at constant pressure.

(ii) Given that an ideal gas goes through the following changes as indicated in the diagrams below.



Prove that coefficient of expansion of an ideal gas at constant volume **($\propto_v$)** is equal to coefficient of expansion of an ideal gas at constant pressure **($\propto_p$)**.

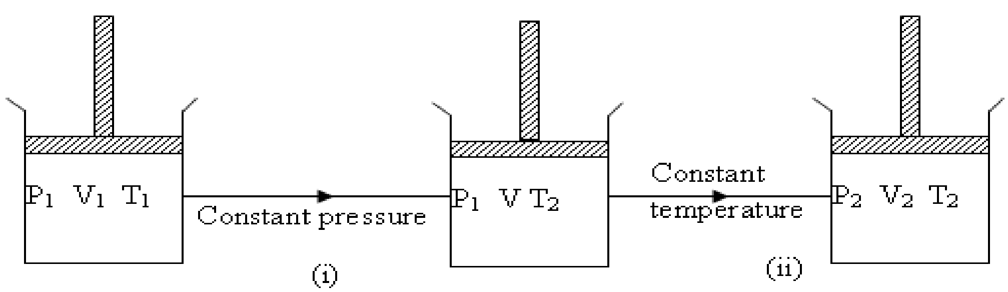
(b) (i) State pressure law of ideal gases.

(ii) Use kinetic theory to account for Boyle's law of ideal gases.

(iii) Describe an experiment to verify Charles's law of ideal gases.

(c) The density of oxygen at s.t.p is $1.43 kg/m^3$. A 20 litre cylinder is filled with oxygen at 25 atmospheres and temperature 27°c, what is the mass of oxygen in the cylinder.(Hint, Assume 0_2 to be ideal)

4) (a) (i) Use the following changes gone through by a fixed mass of a gas to derive the equation of state.



(b) A balloon of weight 0.8N is filled with helium at 10°c and 2.5 atmospheres. The balloon contains 2.3moles of helium. Find;

(i) The volume of helium in the balloon.

(ii) The extra weight that the balloon can lift at the earth's surface.(molar masses of helium and air in which the balloon floats are 4.0g and 29.0g respectively) (Hint, Assume He to be ideal).

c) (i) A fixed mass of the has a volume of $200m^3$ at 57℃ and 780mmHg. Find its volume at s.t.p.

(ii) A cylinder contains 2.0kg of nitrogen at a pressure of $3.0x10^6$pa and 17.9℃ determine its mass at s.t.p i.e. at 0℃ and $1.01x10^5$pa.

Behavior of gases

A gas is a state of a substance above its critical temperature.

Gases are either ideal/perfect or real for instance;

- Ideal gases (perfect gas) are those which obey Boyle's law of perfect gases at all temperatures and pressure with negligible intermolecular forces of attraction and negligible size of each of their molecules thus occupy negligible volume of the container the gas occupies.
- Real gases are those which only obey Boyle's law of perfect gases only at temperatures greater than the critical temperature and pressure less than the critical pressure.
  This is because their intermolecular forces of attraction and volume of their molecules compared to that of the container are not negligible.

Comparison of gases

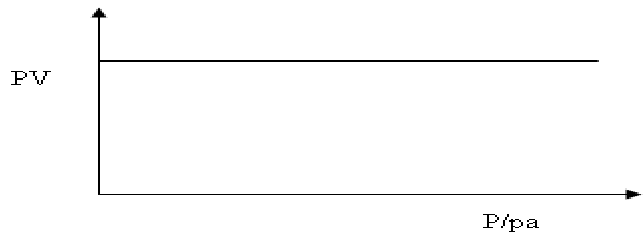
Ideal (perfect gas).	**Real.**
Their intermolecular forces of attraction are negligible.	Their intermolecular forces of attraction are not negligible.
The size of the molecules is negligible thus occupy negligible volume compared to that of the container the gas occupies.	Molecules have a noticeable volume thus occupy a noticeable volume.
Their internal energy is independent of their volume but only depends on temperature.	Their internal directly depends on both their volume and temperature.
They obey Boyle's law of perfect gases at all temperature and pressure.	They obey Boyle's law of perfect gases at temperature>T_c and pressure $P<P_c$.

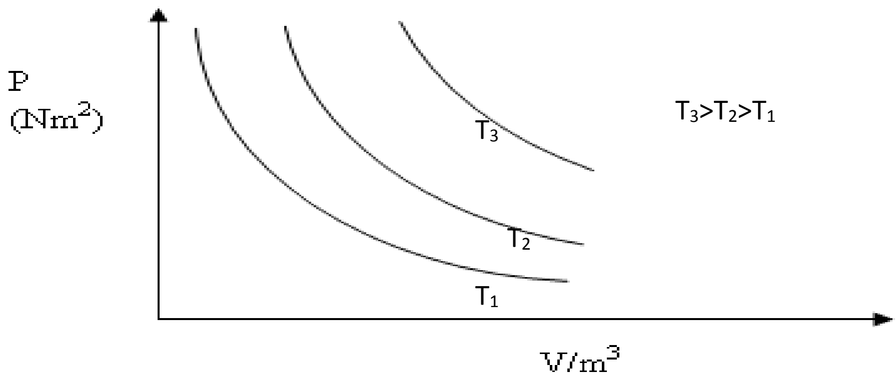
Isothermals of gases

An **isothermal** of a gas is the behavior of a gas at constant temperaturefor instance**;**

Isothermals for ideal gases

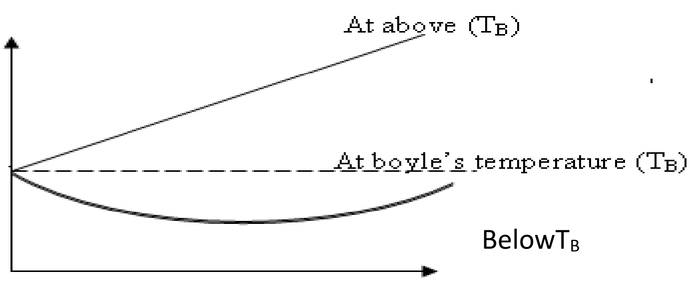
By Boyle's law, PV=constant implying that PV is independent of pressure such that the **PV** against **P or P against V** for an ideal gas are as shown below.





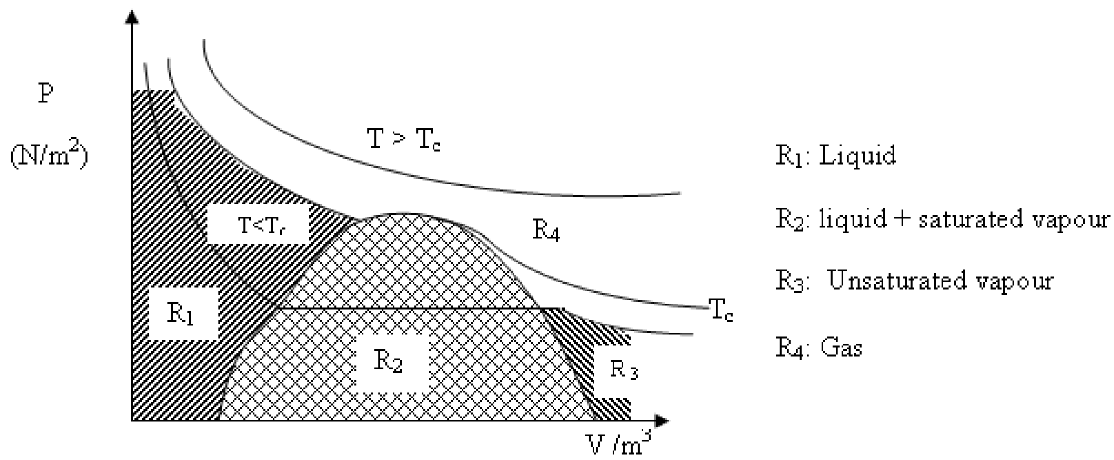
Isothermals for real gases

(i)A plot of PV against P is as shown below



- At $T<T_B$, intermolecular forces of attractions of a real gas out weigh its intermolecular forces of repulsion such that gas molecules are less free to move randomly within the container in which the gas is trapped. Due to this restricted motion, gas molecules exert less pressure than what an ideal gas would exert thus reduction in PV at $T<T_B$.
- At $T>T_B$, intermolecular forces of repulsion of a real gas out weigh its intermolecular forces of attraction such that gas molecules are more free to move randomly within the container in which the gas is trapped. Due to this less restricted motion, gas molecules exert much more pressure than what an ideal gas would exert resulting into increased PV at $T>T_B$.

*(ii)The **P** against **V** is as shown below*



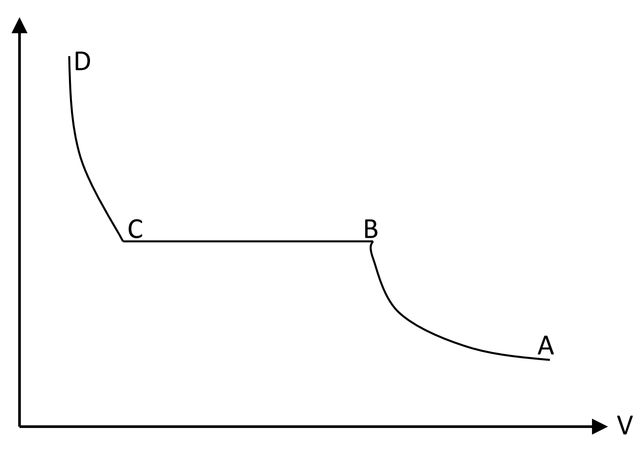
Explanation:

At $T>T_c$ the gas obeys Boyle's law since at high temperatures ($T>T_c$), intermolecular forces of real gases are weakened thus negligible.

Compressing the gas at$T<T_c$, volume reduces at constant pressure due to condensation of the gas until it is completely liquid in region R_1.

The sharp rise in pressure with small change in volume in region R_1 is due to the fact that liquids are difficult to compress unlike gases.

Pressure against volume curve for a real gas undergoing compression below its critical temperature.



In region AB, there is unsaturated vapour, which fairly obeys Boyle's law.

In BC, the vapour is saturated, the pressure remains constant as the volume reduces and a liquid is formed.

In CD, all the vapour has turned into liquid, there is a small change in volume for large pressure increase.

Behavior of real gases

When deriving the equation of state (PV=nRT) we assume that;

- The intermolecular forces of attraction between molecules are negligible.
- The volume of the molecules is negligible compared to the entire volume the container occupied by the gas.

  However, these assumptions hold for real gases only at
- Pressures < P_c (critical pressure).
- Temperatures above T_c (critical temperature).

  Therefore a real gas is one which does not obey ideal gas laws at temperatures <T_c and pressure >P_c that is;

  Its intermolecular forces of attraction and volume occupied by gas molecules compared to the volume of the container occupied are not negligible

Kinetic theory of gases

The kinetic theory is based on the following fundamental assumptions:

- Molecules of an ideal gas move randomly and continuously make perfect elastic collisions with themselves and the walls of the container (i.e. kinetic is conserved).
- Intermolecular forces (both attractive and repulsive) are negligible.
- The volume occupied by the gas molecules is negligible compared to that of the container implying that they are free to move randomly in the container.
- The duration of collision is negligible compared to the time between collisions i.e. the time the molecules spend in contact with the walls is negligible compared to that a molecule takes to move to the opposite wall and back.
- The total number of molecules is large.

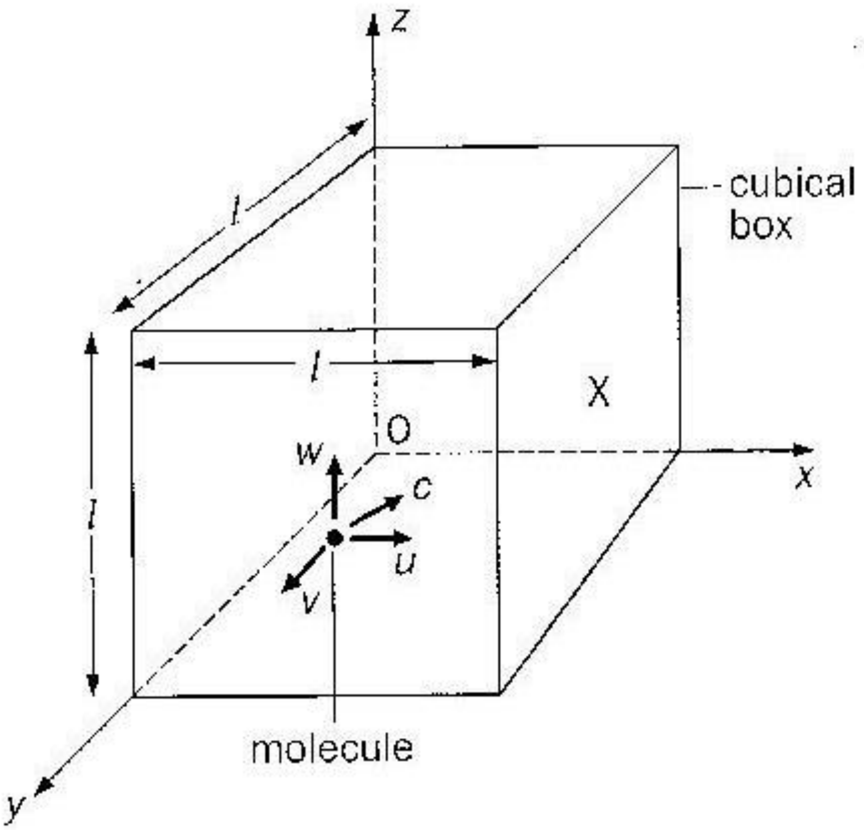
NB: these assumptions are made when deriving the expression for pressure.

Kinetic theory and gas pressure

A gas exerts pressure to the walls of the container it occupies because of their continuous bombardment with the walls of the container. When the gas molecules hit the walls at right angles their momentum is reversed. The rate of change of this momentum results into a force exerted on the walls and the pressure exerted is this force per squared meter.

Expression for gas pressure on basis of kinetic theory (derivation of P =$\frac{1}{3}\rho\overline{c^2}$

Consider a cube of sides L containing ***N identical*** molecules of the same gas each of mass m.



When a molecule moving along **x**with velocity $u_1$collides with face A **normally** its momentum changes such that the resulting change in momentum is = $mu_1 - (-mu_1) = 2mu_1$.
Since the gas is assumed to make elastic collisions with the wall, the duration of collision is negligible compared to the molecule's time between its collisions i.e. t $=\frac{2L}{u_1}$

The force this molecule exerts on the wall = the rate of change of momentum $=\frac{2mu_1}{t} = 2mu_1 \times \frac{u_1}{2L} = \frac{mu_1{}^2}{L}$

Pressure exerted on this wall by this molecule $P = \frac{1}{L^2} \times \frac{mu_1{}^2}{L} = \frac{mu_1{}^2}{L^3}$.

Since the gas is assumed to have negligible intermolecular forces of attraction, pressure exerted on this wall by N identical molecules $P_N = \frac{m}{L^3}(u_1{}^2 + u_2{}^2 + --- + u_N{}^2)$.

If u^2 is the mean square speed of N molecules along **x** then; $u^2 = \frac{(u_1{}^2 + u_2{}^2 + --- + u_N{}^2)}{N}$.

$\Rightarrow$ $P_N = \frac{m}{L^3}Nu^2$

Since molecules are identical and are in independent motion, **then,$\boldsymbol{u^2 = v^2 = w^2}$** such that for 3- dimensions, $u^2 = \frac{1}{3}\overline{c^2}$,

Therefore the total pressure P_T due to motion of **N** molecules in 3-dimensions

$P_T = \frac{1}{3L^3}Nm\overline{c^2} = \frac{1}{3}(\frac{Nm}{L^3})\overline{c^2}$, but $\frac{Nm}{L^3} = \rho, thus\ P_T = \frac{1}{3}\rho\overline{c^2}$

Root- mean square speed ($c_{r.m.s}$)

Is the square root of the mean of the squares of the velocitiesof the molecules.It is given by $c_{r.m.s}= \sqrt{\overline{c^2}} = \sqrt{\frac{3P}{\rho}}$

Mean square speed is the average square speed of the gas molecules at a particular temperature "T" i.e. $<c^2> \alpha T$.

Example

1. Calculate the root mean square speed of the gas molecules at 0^0C and a pressure of 1.0atmosphere. The density of hydrogen at o^0C is 0.09kgm^{-3}.
2. Calculate the speed of sound in the atmosphere of Jupiter at temperature of -130°C and mainly composed of methane gas of mass 6.04g. Assume that the speed of sound is 0.682 times the r.m.s speed of one mole of methane.

   Solutions:

   From $PV = \frac{1}{3}Nm < c^2 >= nRT$. For one mole $N=N_A$.

   $< c^2 >= \frac{3nRT}{Nm}$

   $< c^2 >= \frac{3*1*8.314*(-130+273)}{6.02*10^{23}*6.04*10^{-3}} = 9.8*10^{-19}m^2/s^2$

   $\therefore$ The speed of sound $= 0.682*\sqrt{9.8*10^{-19}} = 6.75*10^{-10}m/s$
3. Calculate the root mean square speed of the hydrogen molecules at 27°C, given that the density of hydrogen at 1.0×10^5N/m^2 and temperature 0°C is 0.09kg/m^3.

   Solutions:

   From $P = \frac{1}{3}\rho < c^2 >$. For $P=1.01\times10^5$Pa, T=273K

   $< c^2 >= \frac{3*1.01*10^5}{0.09} = 3.3*10^6 m/s$

   Now using $< c^2 >\propto T$

   $\frac{3.3*10^6}{< c^2 >} = \frac{273}{300}$

   $< c >= \sqrt{\frac{3.3*10^6*300}{273}} = 1.9*10^3 m/s$

4. (a) At a certain time speeds of seven molecules are as follows:

Number of particles n	1	3	1	1	1
Speed/ms^{-1}	2.0	3.0	4.0	7.0	5.9

Calculate the root mean square speed of the particles.

Solutions:

$<c^2> = \frac{(1x4)+(3x9)+(1x16)+(1x49)+(1x34.81)+}{7}$ **≈18.7m^{2}s^{-2}**

<c> ≈4.3ms^{-1}

(b) (i) Calculate the root mean square speed of air molecules of density 1.56kgm-3 in a container at a pressure of 1.38x10^{5}pa.

Solutions:

$<c> = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3x1.38x10^5}{1.56}}$ **≈515.2ms^{-1}**

(ii) Calculate the temperature of oxygen gas at which its root mean square speed is twice its root mean square speed at 59.3℃.

Solutions:

$\frac{<c_1^2>}{<c_2^2>} = \frac{T_1}{T_2}$, T_1= 4x(59.3+273) =1329.2K.

Deductions from P $=\frac{1}{3}\rho\overline{c^2} = \frac{1}{3}\left(\frac{mN}{V}\right)\overline{c^2} = \frac{1}{3}\overline{c^2}\left(\frac{M}{V}\right)$

a) Kinetic energy of a gas

Taking PV =⅓Nm$<\overrightarrow{c^2}>$…. (i) and that PV =nRT = $\frac{N}{N_A}RT$……(ii)

$\Rightarrow$ ⅓Nm$<\overrightarrow{c^2}>$ = $\frac{N}{N_A}RT$

$\Rightarrow$⅓N_Am$<\overrightarrow{c^2}>$ =RT …….. (iii)

But since $N_A m$ is the total Molar mass **M** then$\frac{2}{3}\left(\frac{1}{2}N_A m<\overrightarrow{c^2}>\right)$=RT=$\frac{2}{3}(k.e)$

$\Rightarrow$ The total kinetic energy of the gas at temperature T is k.e. =$\frac{3RT}{2}$ implying that k.eαT.

From equation (iii) we deduce that kinetic energy of a single molecule (k.e)' =$\frac{3}{2}\left(\frac{RT}{N_A}\right)$ = $\frac{3}{2}k_BT$

where **k_B** is called the Boltzmann constant i.e. $k_B=\frac{R}{N_A}=\frac{8.314}{6.02x10^{23}}$=1.381x10^{-23}JK^{-1}.

Example

1. A mole of an ideal gas at 300K is subjected to a pressure of 1.0x10^{5}Pa such that its volume changes to 0.025m^3. Find
(i) Molar gas constant.
(ii) Boltzmann constant.
(iii)The average translational kinetic energy of a single molecule of the gas

Solutions:

(i) $R = \frac{PV}{T} = \frac{1*10^5*0.025}{300} = 8.33JK^{-1}mol^{-1}$

(ii) $k_B = \frac{R}{N_A} = \frac{8.33}{6.02*10^{23}} = 1.38*10^{-23}\,JK^{-1}$

(iii) $k.e = \frac{3}{2}k_B T = \frac{3}{2}\left(1.38*10^{-23}\right)*300 = 6.21*10^{-21}\,J$

b) Avogadro's hypothesis

Avogadro's hypothesis states thatequal volumes of ideal gases at the same temperature and pressure contain the same number of molecules/moles.

Proof.

Consider two different gases of equal volumes and at the same temperature and pressure.

For gas 1, $P_1V_1 = ⅓N_1m_1 < \overrightarrow{c_1^2} > = \frac{N_1RT_1}{N_A} = N_1k_BT_1$…. (i)

For gas 2, $P_2V_2 = ⅓N_2m_2 < \overrightarrow{c_2^2} > = \frac{N_2RT_2}{N_A} = N_2k_BT_2$…. (ii)

⇒ Since these gases have equal volumes and are at the same temperature and pressure we get that $N_1k_BT = N_1k_BT$thus $\mathbf{N_1=N_2}$**.**

c) Graham's law of diffusion of gases

For an ideal gas its rate of diffusion (speed) is inversely proportional to the square root of its density at constant pressure and temperature i.e. rate of diffusion$\boldsymbol{\alpha}\frac{1}{\sqrt{\rho}}$

Proof.

For gas 1 $\mathbf{P_1} = ⅓\boldsymbol{\rho_1} < \overrightarrow{c_1^2} > \Rightarrow < \overrightarrow{c_1^2} > = \frac{3P}{\rho_1}$ ….(i).

For gas 1 $\mathbf{P_2} = ⅓\boldsymbol{\rho_2} < \overrightarrow{c_2^2} > \Rightarrow < \overrightarrow{c_2^2} > = \frac{3P}{\rho_2}$ …..(ii).

Taking (i)/(ii) yields $\frac{<\overrightarrow{c_1^2}>}{<\overrightarrow{c_2^2}>} = \frac{\rho_2}{\rho_1} \Rightarrow \frac{Rate\ of\ gas\ 1}{Rate\ of\ gas\ 2} = \sqrt{\frac{\rho_2}{\rho_1}}$.

Boyle's law of perfect gases

Pressure of a fixed mass of a perfect gas is inversely proportional to its volume at constant temperature.

Proof.

From $PV = ⅓Nm < \overrightarrow{c^2} > = \frac{2}{3}\left(\frac{1}{2}Nm < \overrightarrow{c^2} >\right) = \frac{2}{3}(k.e)$… (i).

$\Rightarrow PV = \frac{2}{3}(k.e)$.

Since (k.e)T yet with Boyle's law T is constant, it implies that for a fixed mass of an ideal gas

$PV = constant$

d) Charles's law of perfect gases

Volume of a fixed mass of an ideal gas is directly proportional to its temperature at constant pressure.

Proof.

From PV =⅓Nm$< \overrightarrow{c^2} >$ = $\frac{2}{3}\left(\frac{1}{2}\text{Nm} < \overrightarrow{c^2} >\right)$ =$\frac{2}{3}(k.e)$…..(i)

But k.e. =$\frac{3RT}{2}$……(ii)

Substituting for k.e. =$\frac{3RT}{2}$ into equation (i) we get PV =$\frac{2}{3}\left(\frac{3RT}{2}\right)$=RT ⇒ V=$\left(\frac{R}{P}\right)T$.

Therefore $\frac{V}{T}$ = constant as Charle's law states.

e) Dalton's law of partial pressures

The total pressure of a mixture of gases which do not chemically mix at constant temperature is equal to the sum of the partial pressures of the individual gases that constitute the mixture. The term **partial pressure** refers to the pressure a gas would exert if allowed to fill the entire volume initially occupied by mixture of gases which do not chemically mix at constant temperature.

Proof

From PV =nRT= N$\left(\frac{R}{N_A}\right)T$=N$k_B$T.

For a gas of N_1 molecules occupying a container of volume V we have that $P_1V = N_1k_BT$… (i)

For a gas of N_2 molecules occupying a container of volume V we have that $P_2V = N_2k_BT$…(i)

When a mixture of these gases occupy the same container we have that $P_TV = (N_1+N_2)k_BT$…(iii).

This implies that (i) + (ii) = (iii).

$(P_1+P_2)V = P_TV$ ⇒$\mathbf{P_T = P_1 + P_2}$**.**

Examples

1. Two gas bulbs of volume 200cm^3 and 100cm^3 respectively, are connected by a capillary tube. The apparatus is hermetically sealed and contains air at a pressure of 760mmHg and 15^{o}C. The 200cm^3 bulb is immersed in steam at 100^{o}C and the other remaining at 15^{o}C. Find the new pressure of the air in the tube.

   Solutions:

   Using PV=nRT

   At 15^{o}C

   $$n = \frac{PV}{RT} = \frac{76*(200+100)*10^{-6}}{R*288} \text{ ………… (i)}$$

   After raising the temperature to 100^{o}C and the other left at 15^{o}C, then

   $$n_1 = \frac{P_2*200}{R*373}, \text{ and } n_2 = \frac{P_2*100}{R*288}$$

   But the total number of moles remains the same i.e. $n_1+n_2=n$

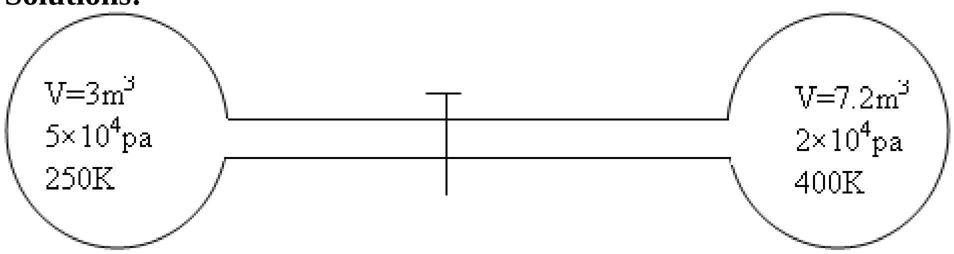
$$\frac{76*300}{288*R} = \frac{200*P_2}{373*R} + \frac{100*P_2}{288*R}$$

$$\frac{76*300}{288} = \frac{200*P_2}{373} + \frac{100*P_2}{288}$$

$$P_2 = \frac{8504400}{(200*288+37300)} = 89.6cmHg$$

2. Two cylinders "A" and "B" containing an ideal gas are connected by a tap having a negligible volume. "A" contains $3m^3$ of the gas at 250K and 5×10^4Pa while "B" contains $7.2m^3$ of the gas at 400K at 2.0×10^4Pa. Find the pressure after the connecting tap has been opened and equilibrium has been reached, assuming that the temperature of "A" and "B" remains constant.

Solutions:



At equilibrium before opening the tap $n_T=n_A+n_B$.

From $PV = nRT$, $n = \frac{PV}{RT}$

$$\Rightarrow \frac{P_A V_A}{RT_A} + \frac{P_B V_B}{RT_B} = n_T$$

$$n_T = \frac{5*10^4*3}{250R} + \frac{2.0*10^4*7.2}{400R}$$

On opening the tap, gases escape from A to B (since it's at a higher temperature) until similar pressure is achieved in both containers i.e.

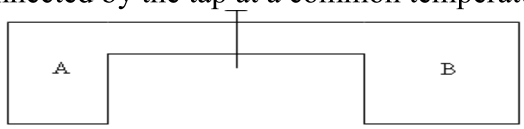
$$n_T^1 = n_A^1 + n_B^1 = P\left(\frac{3}{250R} + \frac{7.2}{400R}\right)$$

But these changes do not alter n, i.e. $n_T = n_T^1$

$$\therefore P\left(\frac{3}{250} + \frac{7.2}{400}\right) = \left(\frac{5*10^4*3}{250} + \frac{2*10^4*7.2}{400}\right)$$

$$P = 3.22*10^5 Pa$$

3. Two cylinders A and B of volumes V and 3V respectively are separately filled with a gas and connected by the tap at a common temperature "T" as shown below.



The pressure of the gas in A and B are P and 4P respectively. When the tap is opened, the cylinders' equilibrium pressure is found to be 60Pa. Find the value of P.

Solutions:

Before opening the tap, at equilibrium $n_T = n_A + n_B$

$n_T = \frac{PV}{RT} + \frac{12PV}{RT} = \frac{13PV}{RT}$…… (i)

After opening the tap, equilibrium is attained when both cylinders are at the same pressure.

$n'_T = \frac{60V}{RT} + \frac{180V}{RT} = \frac{240V}{RT}$……. (ii)

Since the number of moles do not change, then;

$\frac{13PV}{RT} = \frac{240V}{RT}$…….(iii) From which P ≈18.5pa.

4. Two hollow spheres A and B of volume 500cm^3 and 250cm^3 respectively are connected by a narrow tube fitted with a tap. Initially, tha tap is closed and A is filled with an ideal gas at 10^{0}C at a pressure of 3.0x10^{5}Pa `and B is filled with an ideal gas at 100^{0}C at a pressure of 1.0x10^{5}Pa. calculate the:

(i) Equilibrium pressure when tha tap is opened.

(ii) Resulting temperature when the tap is opened.

Solution

(i) Initially: using PV = nRT, $n = \frac{PV}{RT}$, For A: $n_A = \frac{3.0\times10^5\times500}{283R}$

For B: $n_B = \frac{1.0\times10^5\times250}{373R}$

Finally: for A, $n_A^! = \frac{P_A\times750}{283R}$, and for B: $n_B^! = \frac{P_B\times750}{373R}$,

Therefore, the number of moles in each sphrere remains un changed.

Hence: $n_A = n_A^! \Longrightarrow \frac{3.0\times10^5\times500}{283R} = \frac{P_A\times750}{283R} \Longrightarrow P_A = 2\times10^5 Pa$

Also: $n_B = n_B^! \Longrightarrow \frac{1.0\times10^5\times250}{373R} = \frac{P_B\times750}{373R} \Longrightarrow P_B = 33.\times10^4 Pa$

Equilibrium pressure is therefore the total pressure, hence

Total pressure = $P_A + P_B = 2\times10^5 + 3.3\times10^4 = 2.33\times10^5 Pa$

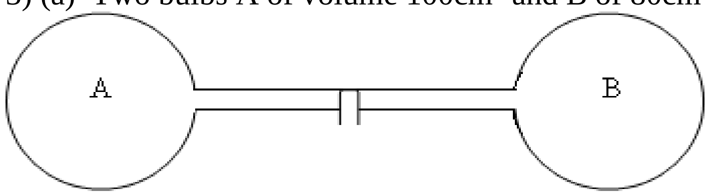
(iii) for resulting temperature, using PV = nRT where n= $n_A + n_B$, thus $\frac{3.0\times10^5\times500}{283R}+\frac{1.0\times10^5\times250}{373R}=\frac{2.33\times10^5\times750}{RT}\Rightarrow T=292.7K$

Assignment

1. A vessel of volume $1.0\times10^{-3}m^3$ contains helium at a pressure of 2.0×10^5Pa and temperature of 300K.
   (i) What is the mass of helium in the vessel?
   (ii) How many helium atoms are there in the vessel?
   (iii) Calculate the r.m.s speed of the helium atoms (Molar mass of helium is 4.0g)

2. Calculate the root mean square speed of the molecules of an ideal gas at 127°C given that the density of the gas at a pressure of 1.01×10^5Pa and temperature of 0°C is $1.43kg/m^3$.

3) (a) Two bulbs A of volume $100cm^3$ and B of $80cm^3$ are connected as shown below



The connecting tube is of negligible volume. Initially **"A"** is filled with ideal gas at 20℃ at 3.5×10^5Pa and **"B"** is filled with an ideal gas at 100℃ at 1.2×10^5Pa.

(i) Calculate the equilibrium pressure inside the bulbs when temperature of **A** is maintained at 20℃ and that of B is reduced to 65℃.

(ii) Calculate the equilibrium pressure inside the bulbs when temperature of both **A** and **B** maintained at 54℃.

4) (a) (i)Define an ideal gas.
(ii)Distinguish between ideal gases and real gases.
(iii) State the conditions under which a real gas behaves like an ideal gas.
(b)(i) Sketch a P versus V isothermals of a real gas below and above its critical temperature.
(ii) Account for the isothermal at $T > T_C$ in b(i) above.
(iii) Explain the features of the isothermal of a real gas being compressed below its critical temperature as shown in 4b (i) above.
c) (i) Sketch PV against P isothermal of a real gas below above its Boyle temperature.
(ii) Account for the isothermals drawn in 4c (i) above.

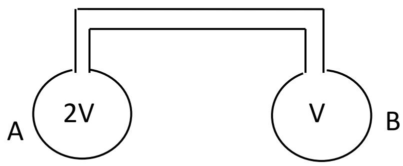
5) (a) (i)Define a real gas.
(ii)State four major assumptions made when deriving $P=1/3\rho< C^2>$ of a gas.
(Hint: The terms take their usual meanings)

(iii) Modify two of the assumptions stated in a(ii) to cater for real gases.

(iv) Explain how these modified assumptions in a(iii) affect the ideal gas equation.

(b) (i) State and derive Dalton's law of partial pressure.

(ii) Define Partial pressure.

c) 

Two vessels **A** and **B** of volume **2V** and **V** respectively are connected by a tube of negligible volume shown above. The vessels contain air of total mass 3.0×10^{-3}kg at 30°C and pressure of 1.0×10^{5} pa. Vessel **A** is cooled to 0°C and vessel **B** is heated to 100°C.Calculate the new equilibrium pressure inside the vessels and mass of the gas in each vessel at this new state.

6) (a)(i) State Avogadro's hypothesis.

(ii) Prove that kinetic energy of an ideal gas molecule is directly proportional to its absolute temperature.

(iii) Calculate the root mean square speed of the hydrogen molecules at 30°c given that density of Hydrogen at 1.5×1^{05} pa and 2°c is 0.19kg/m^3.

(b) (i) State Graham's law of diffusion of ideal gases.

(ii) Two gas bulbs of volume 200cm^3 and 100cm^3 respectively are connected by a capillary tube and contain air at 760mmHg and 15℃. The 200cm^3 bulb is immersed in steam and the other left at 15℃. Find the new pressure in the bulbs at equilibrium state.

7) (a) (i) Account for the pressure exerted on the walls of the container occupied by an ideal gas using kinetic theory of ideal gases.

(ii) Sketch PV versus P and P versus V isothermals of an ideal gas.

(b) (i) Derive $P= \frac{1}{3} \rho<c^2>$ for an ideal gas clearly pointing out the assumptions made (Hint: take the terms to take on their usual meanings)

(ii) Use the above expression in a(i) to derive a Avogadro's hypothesis.

(c) The following table shows the distribution of speed of gas particles. Use the information given to calculate root mean square speed of the particles.

Speed/ms-1	10.0	20.0	30.0	40.0	50.0	21.5
Number of particles	1	3	8	5	4	2

Thermal dynamics

This deals with the behavior of a gas enclosed in a container to which heat is supplied or removed.

First law of thermal dynamics

States that when an amount of heat is supplied to a system, some of it is used to increase the internal energy of the system and the rest is used by the system to dio work to the surrounding.

i.e. $\Delta Q = \Delta U + \Delta W$ where ΔU is the change in the gas's internal energy and ΔW is the work done to expand the gas against external pressure.

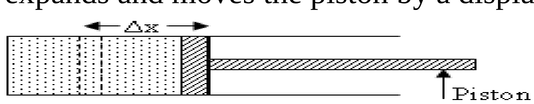
Internal energy of a gas

Real gases have both kinetic energy and potential energy since they have intermolecular forces of attraction and are always in random motion i.e. **ΔU =k.e +p.e**. Kinetic energy is due to the random motion of its molecules and is directly proportional to temperature of the gas yet potential energy is due to the intermolecular forces of attraction which depends on the separation of the molecules.

Since ideal (perfect) gases have negligible intermolecular forces of attraction then their internal energy has no potential component which means that **ΔU** is independent of the gas's volume but only depends on its temperature.

External work done by an expanding gas

We consider a gas confined in a frictionless cylinder by a piston such that when heated, it expands and moves the piston by a displacement **Δx**.



If the gas displaces the piston by a force $\vec{F}$, then the work done during the expansion is **F∆x**.
But if the cross sectional area of the piston is A, then this work done $\Delta W = PA\Delta x$.
Since $A\Delta x$= change in volume ΔV, then $\Delta W = P\Delta V$.
If during expansion, the volume changes from V_1 to V_2, then the total work done is

$$W = \int_{V_1}^{V_2} PdV.$$

Heat capacities of a gas

Heat capacity of a gas is defined as the amount of heat required to raise the temperature of a gas by 1K. Temperature of a gas can be raised when either its volume or pressure is kept constant. There fore;

Heat capacity of a gas at constant volume is defined as the amount of heat required to raise the temperature of a gas by 1K at constant volume.

Heat capacity of a gas at constant pressure is defined as the amount of heat required to raise the temperature of a gas by 1K at constant pressure.

Molar heat capacity/principal heat capacity of a gas at constant volume **"C_v"** is the amount of heat required to raise the temperature of 1mole of a gas by 1K at constant volume. The units are $Jmol^{-1}K^{-1}$.This means that $C_v = \left(\frac{\Delta Q}{\Delta T}\right)_V$ where $\Delta Q = \Delta U \Rightarrow \Delta U = C_v\Delta T$ for one mole and $Q = nC_v\Delta T$ for n moles of a gas.

Molar heat capacity/principal heat capacity of a gas at constant pressure **(C_p)** is the amount of heat required to raise the temperature of 1mole of a gas by 1K at constant pressure. The units are $Jmol^{-1}K^{-1}$. This means that $C_p = \left(\frac{\Delta Q}{\Delta T}\right)_p$ where $\Delta Q = \Delta U + \Delta W$ i.e. $= C_p\Delta T$ for one mole and $Q = nC_p\Delta T$ for n mole of a gas.

Example

1. The temperature of a gas in an expandable container is raised from 0^0C to 80^0C at a constant pressure of $4.0x10^5Pa$. If the total heat added is $5x10^4J$, find the number of moles. Take molar heat capacity at constant pressure to be $29.1Jmol^{-1}K$.(21.5 moles)

Specific heat capacity of a gas is defined as the amount of heat required to raise the temperature of 1kg mass of a gas by 1K.

Specific heat capacity of a gas at constant volume **(c_v)** is the amount of heat required to raise the temperature of 1kg mass of a gas by 1K at constant volume $c_v = \frac{C_v}{Molar\ mass}$ and its units are $Jkg^{-1}K^{-1}$.

Specific heat capacity of a gas at constant pressure **(c_p)** is the heat required to raise the temperature of 1kg mass of a gas by 1K at constant pressure

$c_p = \frac{C_p}{Molar\ mass}$ and its units are $Jkg^{-1}K^{-1}$.

Relation between heat capacities C_p and C_v

Suppose 1 mole of a gas is heated through temperature ΔT at constant volume, then heat required $\Delta Q = C_v\Delta T$, $\Delta W = 0$, since $\Delta V = 0$
Therefore, $\Delta Q = \Delta U + \Delta W$, $\Delta Q = \Delta U = C_v\Delta T$ …………………..1
Suppose the 1 mole of a gas is heated through temperature ΔT at constant pressure; then
$\Delta Q = C_p\Delta T = \Delta U + \Delta W \Rightarrow \Delta Q = C_p\Delta T = C_v\Delta T + P\Delta V$ ………………2
Initially; $PV = RT$ and later $P(V + \Delta V) = R(T+ \Delta T)$
Subtracting; $P(V + \Delta V) - PV = R(T+ \Delta T) - RT \Rightarrow P\Delta V = R\Delta T$ …………….3
Substituting 3 into 2 gives, $C_p\Delta T = C_v\Delta T + R\Delta T \Rightarrow C_p = C_v + R$

$\Rightarrow C_p - C_v = R$, Where "**R**" is called the molar gas constant
Also taking $C_v = Mc_v$ and $C_p = Mc_p$, we can deduce that c_p-c_v =r where $r = \frac{R}{M}$ (Specific heat gas constant)
Example
1) The specific heat capacity of neon at a constant pressure is $1.03\times10^3 Jkg^{-1}K^{-1}$. The molecular mass of neon is 20.2. Find ${C_P}/{C_V}$ for neon ($R=8.314Jmol^{-1}K^{-1}$).

Solutions:

$$C_P = c_P.M = 20.2\times1.03\times10^3 = 20.8Jmol^{-1}K^{-1}$$
$$C_V = C_P - R = (20.8 - 8.314) = 12.6Jmol^{-1}K^{-1}$$
$$\gamma = {C_P}/{C_V} = \left({20.8}/{12.6}\right) = 1.64$$

2) The specific heat capacity of a diatomic gas at constant volume is $0.410kJkg^{-1}K^{-1}$. Calculate specific heat capacity at constant pressure and specific gas constant.

Solutions:

Since the gas is diatomic, γ=1.4
$c_p=\gamma c_v$ =1.4x0.410 =0.574$kJkg^{-1}K^{-1}$
Specific gas constant "r" = c_p-c_v = 0.574-0.410 =0.1640$kJkg^{-1}K^{-1}$
(3) 2.00moles of oxygen at 290K is enclosed in an insulated container by a frictionless piston and pressure outside the cylinder is 0.4Mpa. When 1.16kJ of heat is supplied to this gas, its temperature increases to 310K and volume increases by $0.00083m^3$. Calculate the principal heat capacities of the gas and the universal gas constant.

Solutions:

Heat supplied $=nC_p\Delta T$
$C_p = \frac{1.16x1000}{2x(310-290)}$=29.0$JK^{-1}mol^{-1}$
Work done = $P\Delta V = 0.00083x4.0x10^5 \approx 332J$

ΔU =Heat supplied – work done = 1160-332 =828J

By definition $\Delta U = nC_v\Delta T$

$Cv = \frac{828}{2x(310-290)} \approx 20.7JK^{-1}mol^{-1}$

$R = C_p - C_v = 29.0-20.7 = 8.3JK^{-1}mol^{-1}$

4) A gas is enclosed in a cylinder by frictionless piston of area $100cm^{2.}$. 250J 0f heat is supplied to this gas and it expands against external pressure **of 0.1Mpa** and the piston is displaced by 15.0cm along the cylinder. Calculate external work done by the gas and increase in the gas's internal energy.

Solutions:

Force exerted = $100x10^{-4}x1.0x10^{5}$=1000N, Work done = Fxdistance =$1000x15x10^{-2}$=150J

ΔU = Heat supplied – work done = 250-150 =100J

Thermo dynamic processes

A gas can be made to go through a number of changes by changing its variables like pressure, volume and temperature for example;

Isothermal process is one which involves change in pressure and volume of a fixed mass of a gas at constant temperature i.e. $\Delta Q = \Delta W$ since $\Delta U = 0$ because $\Delta T = 0$. This implies that heat supplied during isothermal process is only used to expand the gas with out increasing its internal energy

Adiabatic process is one which involves a change in volume, pressure, and temperature of a fixed mass of a gas at constant heat i.e. $= \Delta U + \Delta W = 0 \Rightarrow \Delta U = -\Delta W$. This means that during adiabatic expansion, the gas cools because it uses part of its internal energy to do work during expansion against external pressure and on compression the gas's temperature rises.

Isobaric process is one which involves a change in volume and temperature of a fixed mass of a gas at constant pressure i.e. .$\Delta Q = \Delta U + \Delta W = C_v\Delta T + P\Delta V$. This implies that heat supplied during isobaric process is used to both expand the gas against external pressure and to increase its internal energy.

Isovolumetric process is one which involves a change in pressure and temperature of a fixed mass of a gas at constant volume i.e. .$\Delta Q = \Delta U$ since $\Delta V = 0$. . This implies that heat supplied during isovolumetric process is only used to increase its internal energy with out expanding it against external pressure.

NB:

- When a gas is heated at constant pressure, heat supplied is used to raise its internal energy and to do work in expanding the gas against external pressure.
- Meanwhile, if heating is done at constant volume of the gas, heat supplied just raises the temperature of the gas implying that $\mathbf{C_p > C_v}$.

Reversible processes

A reversible process is a change in a state of fixed mass of a gas which can be retraced (made to go in reverse direction) by infinitesimal (infinitely small) changes in its variables. Therefore**;**

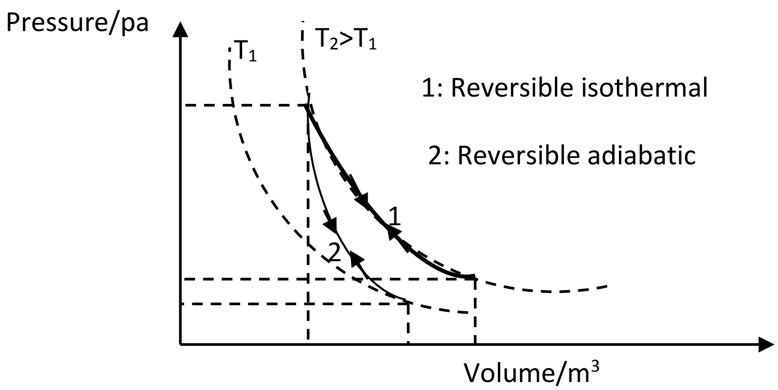
Isothermal reversible process is a change in a state of fixed mass of a gas which can be retraced (made to go in reverse direction) by infinitesimal (infinitely small) changes in its variables i.e. volume or pressure at constant temperature.

Isobaric reversible process is a change in a state of fixed mass of a gas which can be retraced (made to go in reverse direction) by infinitesimal (infinitely small) changes in its variables i.e. volume or temperature at constant pressure

.**Isovolumetric reversible process** is a change in a state of fixed mass of a gas which can be retraced (made to go in reverse direction) by infinitesimal (infinitely small) changes in its variables i.e. temperature or pressure at constant volume.

Adiabatic reversible process is a change in a state of fixed mass of a gas which can be retraced (made to go in reverse direction) by infinitesimal (infinitely small) changes in its variables i.e. volume or pressure or temperature at constant heat.

Isothermals of reversible processes



Isothermal processes

For an isothermal process as defined above Δ Q =PΔV since ΔU =$C_v\Delta$T yet T is constant i.e.. Internal energy is constant. Heat is gained during isothermal expansion and lost during isothermal compression.

This implies that in an isothermal process heat is used to do work in expanding the gas against the external pressure but not to raise its temperature.

Examples of isothermal processes

- Slowly compressing the gas by a light frictionless piston in a highly conducting cylinder.
- The changes in pressure and volume of a frictionless support in a vacuum over which the pendulum bob moves.
- The changes in pressure and volume of sound as its waves pass through vacuum.

Conditions for isothermal process to take place

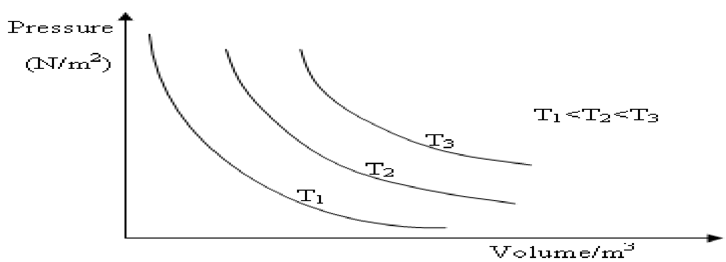
- The gas must be contained in a thin walled and highly conducting vessel such that the rate at heat leaves is equal to the rate at which it leaves to ensure constant temperature.
- The container must be in thermal contact with a constant temperature bath.
- The process must be carried out slowly to allow time for heat exchange.
- The piston must be light in weight and frictionless to eliminate temperature rise due to friction.

NB$_1$:

In practice, it is difficult to achieve an isothermal process because in real sense, heat can not flow through the walls of the vessel unless there is at least a small difference in temperature across the vessel.

This implies that the temperature of the system is bound to fall or rise a little during isothermal expansion or isothermal compression respectively.

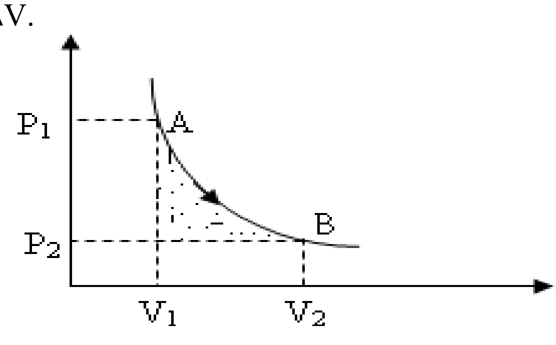
NB$_2$: Isothermal processes obey Boyle's law therefore, their isothermals are as follows.



This implies that for calculations involving isothermal process, we only make use of "**Boyle's law"** i.e. PV =constant. This must be taken as a point of emphasis.

Work done in an isothermal process

Consider an isothermal expansion in which the volume of the gas changes from V_1 to V_2 with an infinitesimal change ΔV.



Work done $=\int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2}\left(\frac{nRT}{V}\right)dV$ =nRTln$\left(\frac{V_2}{V_1}\right)$

NB$_1$: If $V_2>V_1$, the process is an isothermal expansion and work done is positive meanwhile, if $V_1>V_2$, the process is an isothermal compression and work done is negative.

NB$_2$: Since for isothermal process PV=constant (nRT), then

W =nRTln$\left(\frac{V_2}{V_1}\right)$=nRTln$\left(\frac{P_2}{P_1}\right)$ = P_1V_1ln$\left(\frac{P_2}{P_1}\right)$ =P_2V_2ln$\left(\frac{P_2}{P_1}\right)$=$P_1V_1ln\left(\frac{V_2}{V_1}\right)$ = P_2V_2ln$\left(\frac{V_2}{V_1}\right)$.

Adiabatic processes

For an adiabatic process as defined above Δ Q = CvΔT +PΔV =0 since heat Q is constant. This means that for an adiabatic process no heat enters and no heat leaves the container thus ΔU = -ΔW i.e. P =- $C_v\left(\frac{\Delta T}{\Delta V}\right)$. Therefore work done during adiabatic compression is positive and negative for adiabatic expansion.

Examples of Adiabatic processes

The following processes are assumed to be adiabatic because they are fast i.e.

- The warming of a bicycle pump/valve when pumping i.e. pressure, volume and temperature but change in heat is negligible.
- Air rushing out of the valve of the tube of the tyre which has burst.
- The expansion and contraction of air through which a sound wave passes.

Conditions for adiabatic processes

- The gas must be contained in a thick perfectly container such that no heat enters or leaves.
- The process must be carried out quickly/rapidly to minimize time lag for heat exchange.
- The gas must trapped in a perfectly lagged container,
- The container containing the gas must be isolated i.e. no need for constant temperature bath.

Expression related to adiabatic processes

From the definition of adiabatic process;

$C_v\Delta T+P\Delta V= 0$, but PV =RT (for one mole of a gas) $\Rightarrow$ P =$\frac{RT}{V} \Longrightarrow C_V\Delta T + \frac{RT}{V}\Delta V = 0$

Separating variables,

$$C_v\left(\frac{\Delta T}{T}\right)+\left(\frac{R}{V}\right)\Delta V=0 \;\ldots\ldots\; \text{(i)}$$

Integrating equation (i) yields $C_v\int\left(\frac{1}{T}\right)dT+R\int\left(\frac{1}{V}\right)dV = 0$.

$\Rightarrow C_v\ln(T) + R\ln(V)$ =constant ……………….(ii)

For $C_p - C_v = R$, substitute R in (ii) we get, $C_V\mathrm{In(T)} + \mathrm{C_P In(V)} - \mathrm{C_V In(V)}$, thus $C_v\ln\left(\frac{T}{V}\right) + C_p\ln(V)$ = constant, dividing through by C_V

$\Rightarrow\ln\left(\frac{T}{V}\right) + \gamma\ln(V)$ = constant where $\gamma = \frac{C_p}{C_v}$.

Therefore $\ln(TV^{-1}V^{\gamma})$ = constant

$\Rightarrow$**T$V^{\gamma-1}$ = constant…… (ii)**

Substituting for $T = \dfrac{PV}{R}$ into (ii)

$\frac{PVV^{\gamma-1}}{R}$ = c onstant $\Rightarrow$ PV^{γ} = constant……. (iii)

Substituting for $V = \dfrac{RT}{P}$ into (iii)

$P\left(\dfrac{RT}{P}\right)^{\gamma} =$ Constant

$\Rightarrow T^{\gamma}P^{1-\gamma} =$ Constant ………. (iv)

NB:

"γ" is called the coefficient of principal heat capacities of a gas given by; γ $=\frac{C_p}{C_v} = \frac{c_p}{c_v}$.

This coefficient is $\mathbf{\frac{5}{3}}$ **≈1.67**for diatomic gases like; oxygen, carbon dioxide etc and $\mathbf{\frac{7}{5}}$ **= 1.4** for monatomic gases like; hydrogen, helium and carbon monoxide.

As a point of emphasis, it should be noted that for calculations involving adiabatic processes, we only make use of the following equations i.e.

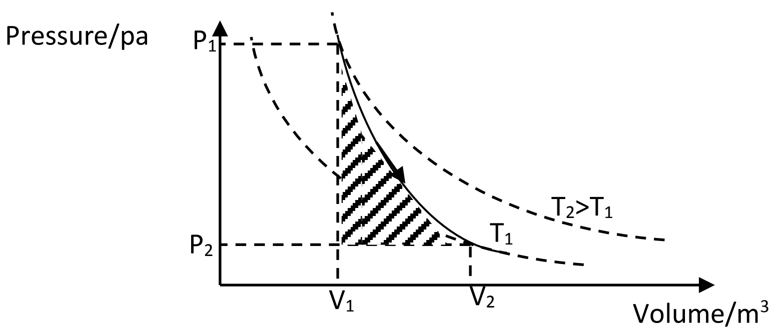
PV^{γ}=constant

$TV^{\gamma-1}$ = constant

$T^{\gamma}P^{1-\gamma}$= constant

Work done by an adiabatic process

Consider an adiabatic expansion illustrated by the following sketch.



Work done = Area under the curve $=\int_{V_1}^{V_2} PdV$

But for adiabatic process $PV^{\gamma} = k$

$\Rightarrow$ W $=k\int_{V_1}^{V_2} V^{-\gamma}dV = \frac{k}{1-\gamma}\left[V^{(1-\gamma)}\right]_{V_1}^{V_2} = \frac{k}{1-\gamma}\left(V_2^{1-\gamma} - V_1^{1-\gamma}\right)$

But $k = P_1V_1^{\gamma} = P_2V_2^{\gamma}$

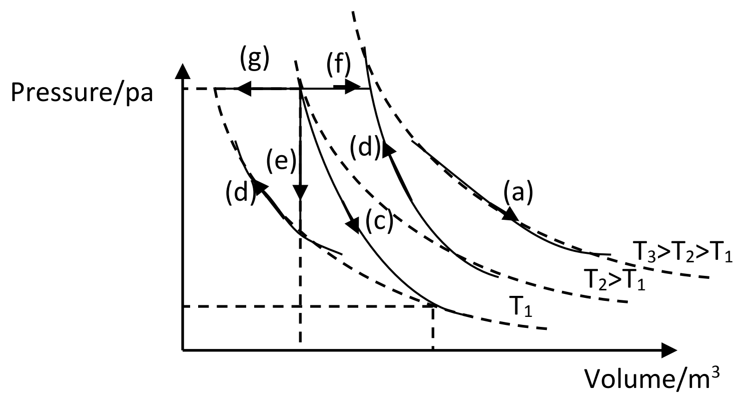
Therefore W $=\frac{1}{1-\gamma}\left(P_2V_2^{\gamma}V_2^{1-\gamma} - P_1V_1^{\gamma}V_1^{1-\gamma}\right) \Rightarrow$ W $=\frac{1}{1-\gamma}(P_2V_2 - P_1V_1)$.

Examples

1) Sketch pressure against volume curves on the same axes for a fixed mass of an ideal gas undergoing through the following changes.

(a)Isothermal expansion (b)Isothermal compression (c) Adiabatic expansion (d) Adiabatic compression (e) Isovolumetric (f) Isobaric expansion (g) Isobaric compression..

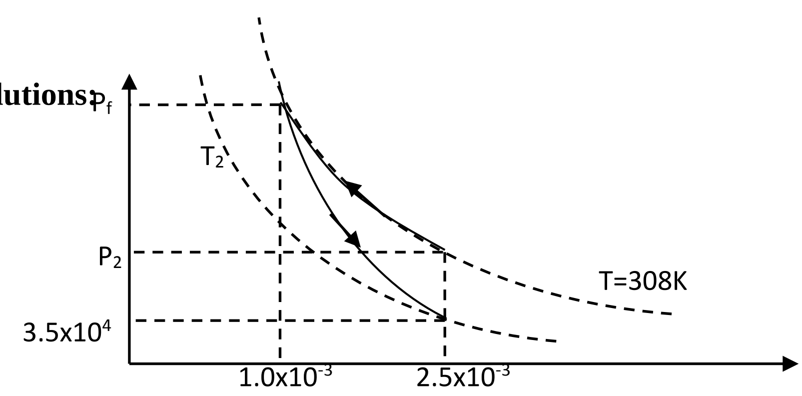
Solutions:



2) A vessel contains $2.5\times10^{-3}m^3$ of an ideal gas at a pressure of $3.5\times10^4 N/m^2$ and a temperature of 35^oC. The gas is compressed isothermally to a volume of $1.0\times10^{-3}m^3$. It is then allowed to expand adiabatically to its original volume $(\gamma = 1.4)$. Find;

i) the final temperature of the gas
ii) the work done in the isothermal compression
iii) final pressure during isothermal compression
iv) final pressure during adiabatic expansion

Solutions:



i) For adiabatic expansion TV^{γ} = constant.

$T2 = 308\left(\frac{1.0}{2.5}\right)^{0.4} \approx 297.3K$

ii) For isothermal compression

$W = nRT\ln\left(\frac{V_2}{V_1}\right) = nRT\ln\left(\frac{P_2}{P_1}\right) = P_1V_1\ln\left(\frac{P_2}{P_1}\right) = P_2V_2\ln\left(\frac{P_2}{P_1}\right) = P_1V_1\ln\left(\frac{V_2}{V_1}\right) = P_2V_2\ln\left(\frac{V_2}{V_1}\right)$.

W =3.5x104x2.5x10-3xln(1.0/2.5) ≈-80.2J

(iii) PV = constant

$3.5x10^4x2.5x10^{-3} = P_2x1.0x10^{-3}$

$P_2 = 8.75x10^4$pa

(iv) PV^{γ}= constant

$Pf = 8.75x10^4x\left(\frac{1.0}{2.5}\right)^{1.4} \approx 2.43x10^4$pa

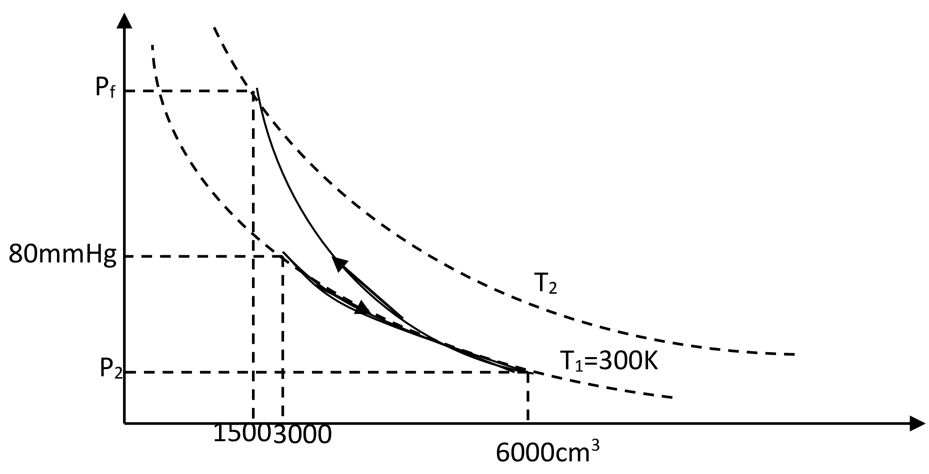
3) A gas at 27°C, volume of 3000cm^3 and a pressure of 80cmHg expands isothermally to double its volume. The gas is then compressed adiabatically to half its original volume.

i) Show the changes on a P-V sketch

ii) Calculate the final temperature and pressure of the gas ($\gamma = 1.4$).

Solutions:

(i)



i) For isothermal expansion PV = Constant.

$P_1V_1 = P_2V_2$

$80 \times V_1 = P_2 \times 2V_1 \Rightarrow P_2 = 40$mmHg.

For adiabatic compression $PV^{\gamma} =$ Constant

$$P_2V_2^{\gamma} = P_fV_f^{\gamma}$$

$$P_f = 80\left(\frac{2V_1}{V_1/2}\right)^{1.4} = 40x(4)^{1.4} = 278.6mmHg$$

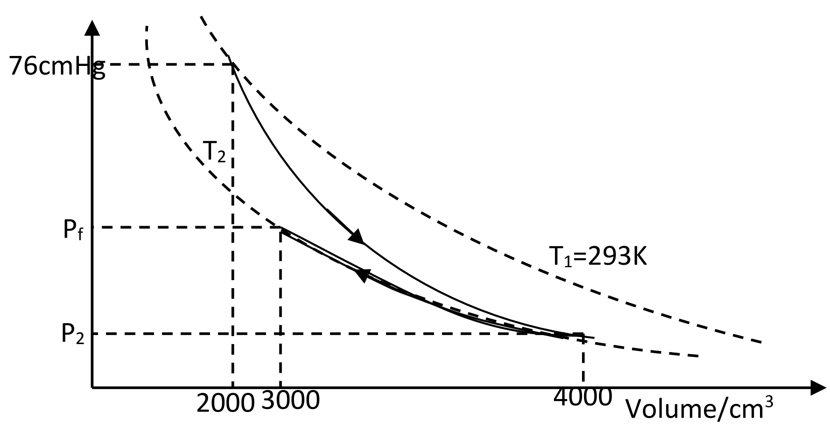
Also for adiabatic compression, $TV^{\gamma-1} =$ Constant

$$T_2V_2^{\gamma-1} = T_fV_f^{\gamma-1}$$

$$\Rightarrow T_2 = 300\left(\frac{2V_1}{V_1/2}\right)^{0.4} = 300(4)^{0.4} = 522.33K$$

4) A mass of air occupying initially a volume of 2000cm^3 at a pressure of 76cmHg and at temperature of 20^{o}C is expanded adiabatically and reversibly to twice its volume and then compressed isothermally and reversibly to a volume of 3000cm^3. Find the final temperature and pressure, assuming the ratio of the molar specific heat capacities of air to be $5/3$.

Solutions:



For adiabatic expansion $P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$$\Rightarrow P_2 = 76 * \left(\frac{2000}{4000}\right)^{1.4} = 76(0.5)^{1.4} = 28.792cmHg$$

For isothermal compression PV = Constant

$\therefore P_2V_2 = P_fV_f \Rightarrow P_f = 76(0.5)^{1.4}\left(\frac{4000}{3000}\right) = 76*(0.5)^{1.4}\left(4/3\right)$ =38.398cmHg.

For adiabatic expansion $TV^{\gamma-1} =$ Constant i.e.

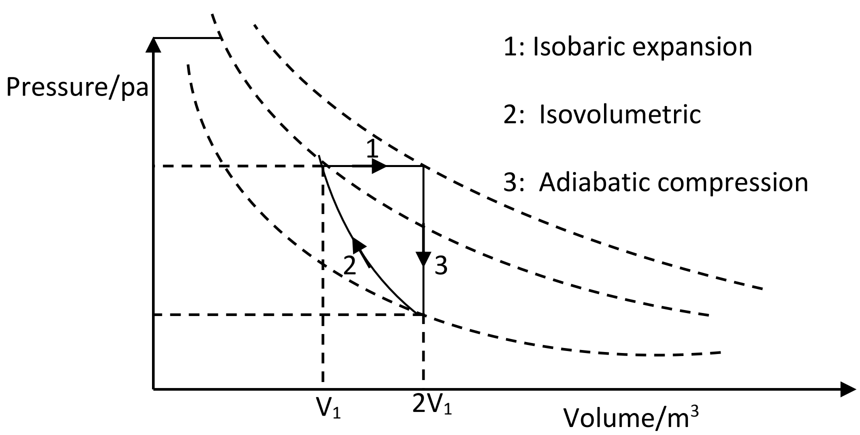
$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$T_2 = 293\left(\frac{2000}{4000}\right)^{0.4} = 293(0.5)^{0.4} = 222.1K$

6) 2 litres of nitrogen at a pressure of 1.0×10^5Pa and a temperature of 27^oC are heated at a constant pressure until its volume is doubled. It is then cooled at constant volume until its pressure is 2.5×10^4Pa. The gas is then compressed adiabatically to its original volume ($\gamma = 1.4$ for nitrogen).

i) Show on a well labeled P-V diagram the above cycle.
ii) Find the final temperature
iii) Pressure of the gas at the end of the compression
iv) If the molar heat capacity of nitrogen at constant pressure is 29.2$Jmol^{-1}K^{-1}$, how much heat is supplied to the gas during the expansion at constant pressure?
v) Find the internal energy generated during the isovolumetric process.

Solutions:



(ii) For isobaric expansion, $\frac{V_1}{T_1} =$ Constant i.e.

$\frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow T_2 = \left(\frac{2V_1}{V_1}\times300\right) = 600K$

For isovolumetric cooling, $\frac{P_2}{T_2} = \frac{P_3}{T_3}$

$$T_3 = \left(\frac{2.5\times10^4}{1.0\times10^5}\right)\times600 = \left(\frac{2.5}{10}\right)\times600 = 150K$$

For the adiabatic compression, $TV^{\gamma-1} =$ Constant

$$T_3V_3^{\gamma-1} = T_4V_4^{\gamma-1} \Rightarrow \left(\frac{2V_1}{V_1}\right)^{0.4}\times150 = (2)^{0.4}\times150 = 197.9K$$

(iii) For adiabatic compression, $PV^{\gamma} =$ Constant

$$P_3V_3^{\gamma} = P_fV_f^{\gamma}$$

$$\Rightarrow P_f = 2.5\times10^4\times\left(\frac{V_1}{2V_1}\right)^{1.4} = 2.5\times10^4(0.5)^{1.4} = 6.6\times10^4\,Pa$$

(iv) Heat supplied $= nC_P\Delta T$

During the isobaric expansion, $\Delta T = (600-300) = 300K$

Also, applying PV = nRT at X $\Rightarrow n = \frac{1.0*10^5*2*10^{-3}}{8.314*300} = 0.08moles$

Heat supplied $= 0.08*29.2*300 = 700.8J$

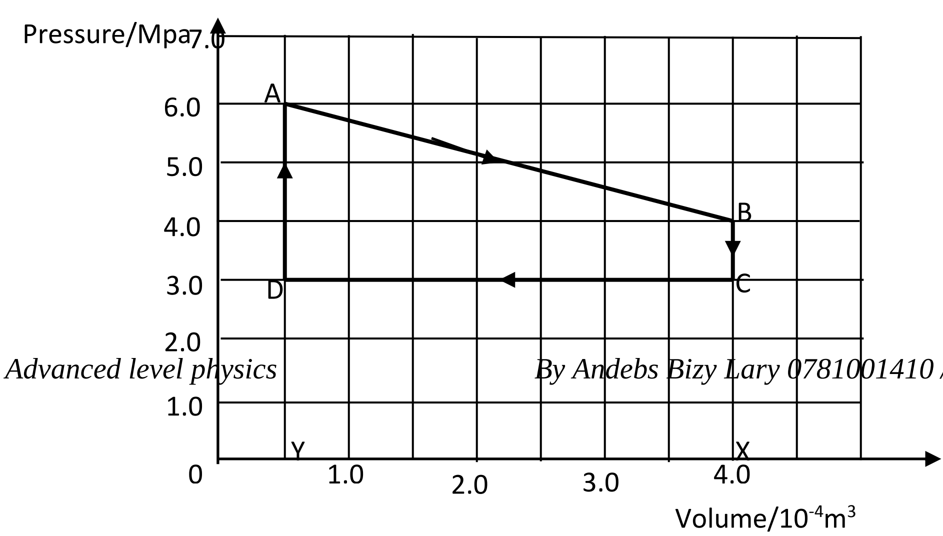
(v) Internal energy generated = heat released $= nC_V\Delta T$

The temperature change ΔT =150-300=-150K

$U = -nC_V\Delta T$

$= 0.08(29.2-8.314)*-150 = -250.632J$

8) A fixed mass of a gas is taken through a closed cycle ABCD of pressure and volume changes as shown in the graph below.



(a) Calculate;
  (i) The work done by the gas on expansion from A to B.
  (ii) The work done by the gas on contraction from C to D.
  (iii) The net work done by the gas during one cycle ABCD.

(b) If the engine rotates at 50cycles per second and it has four cylinders, calculate the power generated by the engine.

Solutions:

(a) (i) Work done =area of ABXY = $\frac{1}{2}$x(AY+BX)xYX = 0.5x(6.0+4.0)x10^6x3.5x10^{-4} =1750J.

(ii) Work done =area of DYXC =3.5x10^{-4}x3.0x10^6 = 1050J

(iii) Net work done = area of ABCD =$\frac{1}{2}$x(AD+BC)xDC= 0.5x(3.0+1.0)x10^6x3.5x10^{-4} =700J

(b) Total cycles made per second = 4x50 =200cyclesper second.

Total power= 200xenergy of one cycle
= 200x700=140Kw.

Test 8

1) An ideal gas of specific heat capacity ratio $\gamma = 1.4$ is expanded adiabatically and reversibly from 70cmHg, volume 4m^3 and temperature of 110^{o}C to a pressure of 34cmHg. It then undergoes a reversible isothermal compression to its original pressure. It is expanded isobarically to its original volume.

Sketch the P-V graphs for the above cycle.

Find;

i) the temperature at the end of the isothermal compression
ii) the volume at the end of the adiabatic expansion
iii) the work done during the isobaric expansion ($W = P(V_2 - V_1)$)
iv) the work done during the isothermal expansion and the adiabatic compression
v) the heat supplied to the gas in the isobaric process ($C_P = 29.0Jmol^{-1}K^{-1}$)

2) Nitrogen gas in an expandable container is raised from 0°C to 50°C at a constant pressure of 4.0×10^5Pa. The total heat added is 3.0×10^4J. Find;

i) the number of moles of the gas

ii) the change in internal energy of the gas ($\Delta U = nC_V \Delta T$)

iii) the work done by the gas ($C_P = 29.1 Jmol^{-1}K^{-1}$)

3) An ideal gas at a pressure of 2.0×10^6N/m^2 occupies a volume of 200cm^3 47.5°C. The gas expands adiabatically to a final pressure of 1.1×10^6N/m^2 ($\gamma = 1.40$, $R = 8.314 Jmol^{-1}K^{-1}$). Find;

i) the number of moles of the gas

ii) the final volume of the gas

iii) the final temperature of the gas

4) A vessel containing 1.5×10^3m^3 of an ideal gas at a pressure of 8.7×10^4Pa and temperature 25°C is compressed isothermally to half its volume and then allowed to expand adiabatically to its original volume ($\gamma = 1.67$). Find;

i) the final pressure and temperature of the gas

ii) sketch the P-V curves for the process

iii) the work done during the isothermal process

5) A cylinder with a piston contains 0.5 moles of oxygen at 2×10^5Pa and 300K. The gas first expands at constant pressure to twice its original volume. It is then compressed isothermally back to its original volume, and finally it is cooled at constant volume to its original pressure.

a) Show the series of processes on a P-V diagram

b) Find;

i) the work done, the heat supplied and the change in internal energy during the initial expansion

ii) the work done during the entire cycle

6) A certain ideal gas has $\gamma = 1.67$.

a) Find its C_P and C_V

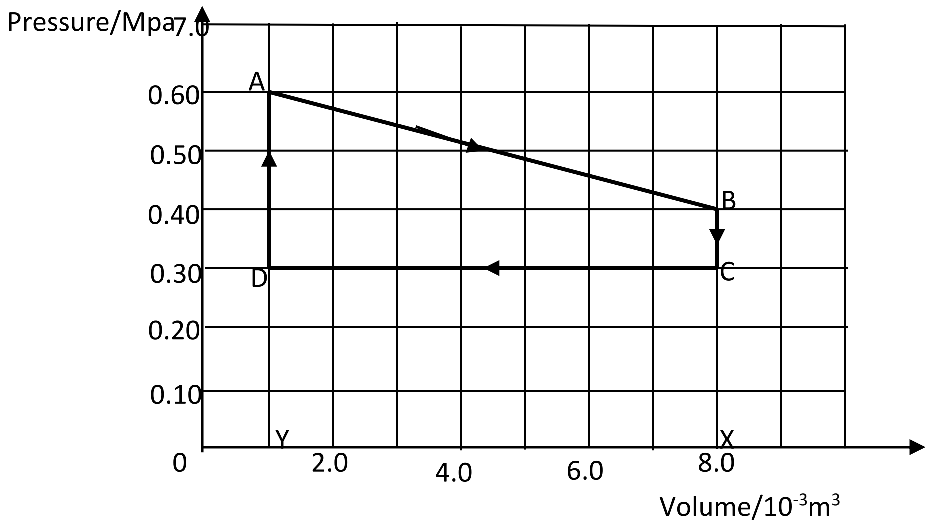
b) A 0.70m^3 of the sample of this gas initially at a pressure of 4.5×10^4Pa is compressed adiabatically to a volume of 0.5m^3. Calculate;

i) the final pressure

ii) the ratio of the final to the initial temperature

iii) the work done by the gas

7) A fixed mass of a gas is taken through a closed cycle ABCD of pressure and volume changes as shown in the graph below.



(c) Calculate;

(iv) The work done by the gas on expansion from A to B.

(v) The work done by the gas on contraction from C to D.

(vi) The net work done by the gas during one cycle ABCD.

If the engine rotates at 50cycles per minute and it has four cylinders, calculate the power generated by the engine

8)(a) Distinguish between;

(i) Reversible isothermal change and a diabetic expansion of gas.

(ii) Isovolumetric change and isobaric change.

(b) Explain the significance of heat supplied to a gas that goes through;

(i) isothermal change

(ii) isovolumetric change

(iii) Isobaric change

(c) Starting from the same point (P_1V_1) sketch P – V curves for;

(i) reversible isothermal

(ii) A diabetic expansion

(d) A fixed mass of a gas in the state (P_1V_1) under goes an isothermal expansion to state (P_2V_2). Obtain the expression for work done by the gas.

(e) Explain why molar principal heat capacity of a solid at constant pressure is less than molar principal heat capacity of a gas at constant pressure.

9) (a) An ideal gas at 2.0 x 10^6pa occupies 2.0 x 10^{-3} m^3 at 47.5°C. The gas expands adiabatically to a final pressure of 110 x 10^5pa $\left(\frac{C_p}{C_v} = 1.4\right)$

Calculate;

(i) Number of moles of the gas
(ii) Final volume of the gas
(iii) Final temperature of the gas($R=8.314Jmol^{-1}K^{-1}$)

(b) Explain why molar heat capacity of a gas at constant pressure is greater than molar heat capacity of a gas at constant volume i.e. $C_P>C_V$

c) Distinguish between;
(i) A reversible process and an irreversible process giving two examples of each.
(ii) An isothermal change and a diabatic change.

(d) Derive the relationship between molar principal heat capacity at constant pressure and molar principal heat capacity at constant volume for one mole of a gas.

10) (a) Explain what happens when a quantity of heat is supplied to a fixed mass of a gas.
(b) A gas initially occupying a volume of 1.0litres at 273K and 1.0 x 10^5pa is compressed isothermally to a volume of 0.5litres. It is then allowed to expand adiabatically to its original volume $\left(\frac{C_p}{C_v} = 5/3\right)$
calculate;
(i) Final temperature and pressure of a gas
(ii) Indicate the process on P.V diagram

(c) A vessel containing 1.5 x 10^{-3} m^3 of an ideal gas at 8.7 x 10^{-2} pa and 25°c is compressed isothermally to half its original volume and then allowed to expand a adiabatically to its original volume
$(\Upsilon = 1.4)$Calculate;
(i) Final temperature and pressure of the gas.
(ii) Sketch the P.V diagram
(iii) Work done during isothermal process.

11) (a) By considering a gas confirmed in a cylinder by a movable piston, use kinetic theory to explain why adiabatic expansion of a gas results into cooling.

(b)A fire extinguisher is filled with 1.0kg of compressed nitrogen gas at a 1.2 x 10^6 pa and 20°C. The gas expands adiabatically to 1.0 x 10^5pa when the nozzle of the extinguisher is opened $(\Upsilon = 5/3)$Calculate;
(i) Original volume of the gas
(ii) Final temperature of gas

(c) Explain why gas A (real gas) and gas B (ideal gas) at a common temperature T, have different amount of internal energies.
(d) State examples of the following;

(i) isothermal changes
(ii) Adiabatic changes

12) (a) Explain why the difference between the principal heats of a solid is small.
(b) (i) Define **principal specific heat capacity** of a gas at a constant pressure.
(ii)Explain why internal energy of an ideal gas Is independent of its volume yet that for real gas depends on its volume.
(iii)Show that work done in expanding the gas of volume V_1 to volume V_2 is given by W = $\int_{V_1}^{V_2} Pdv$where; P is the pressure it exerts on the piston.
(c) Define **isothermal** and **adiabatic** process.
(d) A vessel contains 2.5 x $10^{-3}m^3$ of an ideal gas at 3.5 x 10^4 pa and temperature of 20°C. The gas is compressed isothermally to a volume of 1.0 x 10^{-3} m^3. The gas is allowed to expand adiabatically to its original volume. Given that ϒ = 1.4 , find;

(i) The final temperatures of the gas
(ii) Work done during isothermal compression.

13) (a)(i) State first law of thermal dynamics.
(ii) Define a reversible process.
(b) 2 litres of nitrogen at a pressure of 1.0 x 10^5 pa and temperature of 27°c is heated at a constant pressure until its volume is doubled. It is then cooled at constant volume until its pressure is 2.5 x 10^4pa. The gas is the compressed a adiabatically to its original volume. Given that $\Upsilon = {}^5/_3$. Calculate;

(i) Final temperature of the gas
(ii) Final pressure of the gas
(iii) Supplied heat during the isobaric expansion if C_P= 29.2Jmol^{-1}k^{-1}.
(iv) Internal energy generated during isovolumetric process.

Heat transfer

There are mainly three modes of heat transfer i.e.

- **Conduction** is the transfer of heat from a region of high temperature to region of low temperature without movement of the material medium as a whole.
- **Convection** is the transfer of heat by relative motion of the fluid parts due to differences in density and up thrust caused by temperature differences.
- **Radiation** is the transfer of heat through vacuum i.e. no material medium is required for this transmission.

Mechanism of heat transfer

(a) In good conductor solids.

In these, heat is transferred by the free electrons moving around the whole lattice of the conductor. Such electrons at the hotter end gain more kinetic energy and drift. As they drift, they collide with the neighbouring cooler free electrons. During collision energetic free electrons pass on some of their kinetic energy to the less energetic free electrons thus transferring heat to the cooler end.

(b) In poor conductor solids.

Atoms in a solid are very close together. When one end of the metal is heated, the amplitude of vibration of the atoms there increases since they vibrate more vigorously than before. These atoms will then collide with neighbouring atoms and pass on some of their vibrational energy. This will result into increase in amplitude of the atoms that have taken up some of this energyand they will inturn collide withneighbouring atoms and also passing on some of their vibrational energy. In this waay, heat is propagated towards the colder end.

(c) In liquids.

In these, heat is transferred by molecules locked in their mean positions. Molecules at the hotter end gain kinetic energy and get set into motion, colliding with the cooler molecules (those with less kinetic energy). During collision energetic molecules pass on some of their kinetic energy to the less energetic free molecules thus transferring heat to the cooler region.

(d) In gases.

When a gas is heated, the kinetic energy of the molecules increases which is transferred to other molecules as a result of collision between gas molecules. This energy transfer re distributes in such awayn that all regions eventually contain equal energetic molecules.

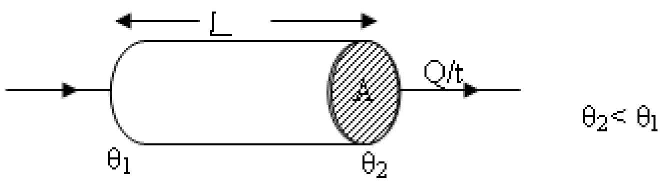
Metals are better conductors of heat than poor conductors because:

i) In metals, ther are free electrons unlike in poor conductors.
ii) On heating one end of a metal, the electrons gain kinetic energy and move faster.
iii) The electrons are lighter, hence covering long distanceand pass on energy quickly though collision with ions on their way towards the cooler end compared to poor conductors eg glass etc.

Thermal conduction

Thermal conduction is the process by which heat flows from a hotter region to the colder region without the net movement of the media itself.

When we consider a slab of material of thickness **L**, cross sectional area A with temperatures θ_1 and θ_2 at its ends; it's the rate of heat flow is $\frac{dQ}{dt} \alpha A\frac{\Delta\theta}{L}$ i.e. $\frac{dQ}{dt}=kA\frac{\Delta\theta}{L}$.......(i)



NB:

Equation (i) is only true for steady state conditions i.e. a conductor is said to be at steady state conditions only if temperatures along its length are steady.

Equation (i) implies that we can not have exchange of heat between bodies at the same temperature.

A larger object transfer much heat than smaller object of the same nature since $\frac{dQ}{dt}$(cross-section area).

From equation (i), we see that factors which affect the amount of heat conducted through a conductor are;

- Area of transmission of the conductor.
- **Temperature gradient** which is the magnitude of temperature difference for every 1 metre length of a conductor at steady state conditions $\frac{\Delta\theta}{L}=\frac{\theta_1-\theta_2}{L}$.
- Thermal conductivity (coefficient of thermal conduction),"k"/nature of the conductor i.e. **Thermal conductivity** is the per squared meter rate of heat flow through a conductor at steady state for every one temperature gradient normal to the surface.

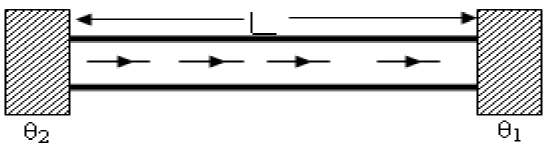
  Thermal conductivity "k" $=\frac{\frac{dQ}{dt}}{A\left(\frac{\Delta\theta}{L}\right)}$ and the units for k are $Wm^{-1}K^{-1}$ and the numerical value is characteristic of a given material.

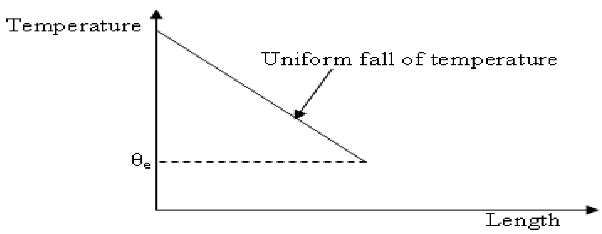
NB: The value "$\frac{k}{L}$" of a material is called its "U" value i.e. $\frac{dQ}{dt}=UA\Delta\theta$. The S.I units of "U" are; $Wm^{-2}K^{-1}$

Temperature distribution along conductors

(a) Lagged conductor/ insulated conductor

In this case, there is no heat loss to the surrounding implying that temperature gradient is uniform.

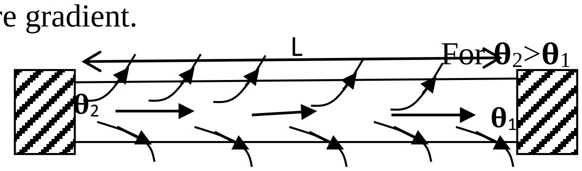




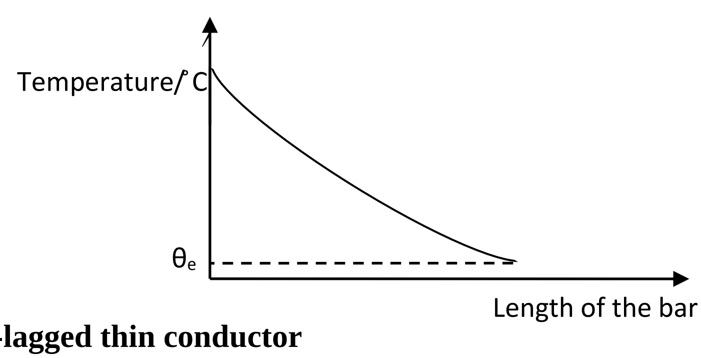
For $\boldsymbol{\theta}_2 > \boldsymbol{\theta}_1$ and θ_e is the body's equilibrium temperature.

(b) Un-lagged thick conductor/ un-insulated thick conductor

Such conductors experience heat losses to the surrounding air by convection resulting into non uniform temperature gradient.

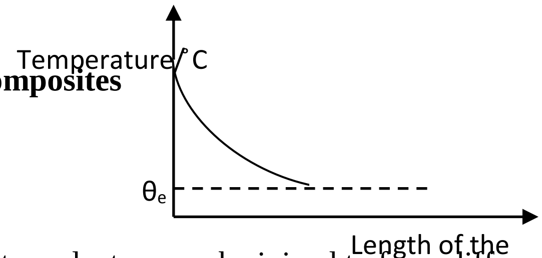


The non uniform temperature gradient is as shown below.



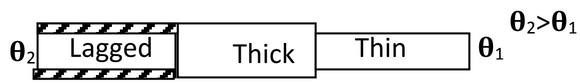
c) Un-lagged thin conductor

Since thin objects loose much heat than thick objects, their temperature fall is steeper than of thick objectsas shown below.

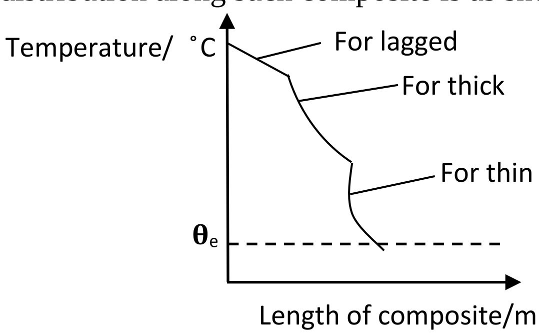


(d) Composites

Different conductors can be joined to form different composites for instance;

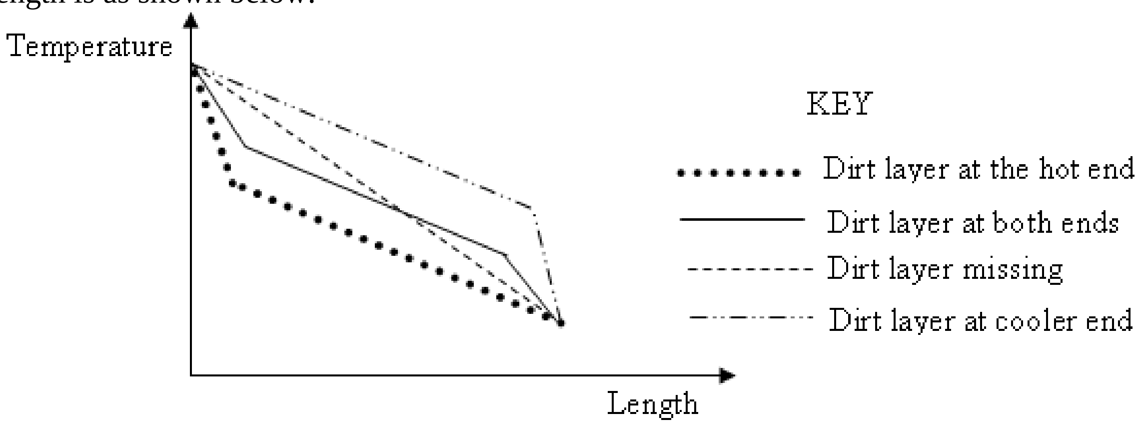


Temperature distribution along such composite is as shown below.



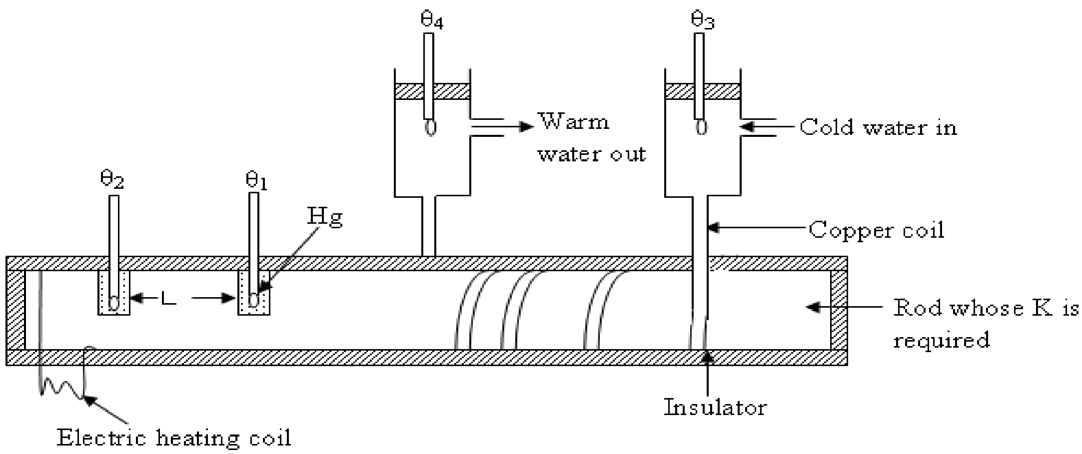
(d) **A thin layer of dirt on a lagged conductor**.

If the lagged conductor contained some thin layer of dirt, the temperature distribution along its length is as shown below.



Determination of thermal conductivity

Good conductor ("Searle's method")



Two holes are drilled in the metal bar at a known distance *l*. the thermometers are put in the holes filled with mercury or oil to increase thermal contact. The diameter d, of the metal bar is measured using micrometer screw gauge and area is calculated. A copper tube is welded around the bar at the cooler end. One end of the bar is heated while the other end is cooledby circulating water in the copper coil from constant head tank. The experiment is left to stand until steady state is reached. The tempearatures $\boldsymbol{\theta_1}$, $\boldsymbol{\theta_2}$, $\boldsymbol{\theta_3}$ and $\boldsymbol{\theta_4}$ are noted. The water of known specific heat capacity C, is collected for a measured time, t, its mass m is calculated. The mass per second is calculated.

The thermal conductivity k is calculated from $k = m'cL\left(\frac{\theta_4-\theta_3}{\theta_2-\theta_1}\right)$

Alternatively

A <u>lagged thin</u> bar of known <u>uniform</u> cross section area **A** is heated at one end with an electrical heater as water is passed through copper tubes rolled over the cold end at a <u>constant rate</u>. The two thermometers which are to give readings **θ_1** and **θ_2** are put in the holes filled with mercury to ensure <u>good thermal contact</u> at a known separation L.
When all the four thermometer readings **θ_1, θ_2, θ_3** and **θ_4** are <u>steady,</u> the mass of water flowing through the copper tube in measured time **t** is determined by using a measuring cylinder.

At this steady state, the quantity of heat flowing through the bar is equal to the heat absorbed by the collected water of mass **m** and of known specific heat capacity **c.**
This implies that $kA\left(\frac{\theta_2-\theta_1}{L}\right) = \frac{m}{t}c(\theta_4 - \theta_3)$ from which $k = m'cL\left(\frac{\theta_4-\theta_3}{\theta_2-\theta_1}\right)$ where $m'=\frac{m}{t}$.

Precautions:

The metal rod should be properly/completely lagged to minimize heat loses to the surrounding.

The two holes for the thermometers should be filled with mercury to minimize error due to poor thermal contacts.

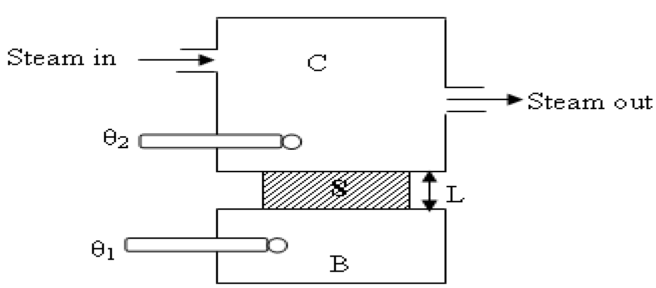
The material should be of uniform cross sectional area to ensure steady states i.e. steady temperatures at all points of the rod.

The rod is made longer than its diameter i.e. rod of small cross sectional area is used to obtain a measurable temperature gradient i.e. since $\frac{\Delta\theta}{L} = \frac{\frac{dQ}{dt}}{kA}$, and for steady state $\frac{dQ}{dr}$ = constant, it implies that $\frac{\Delta\theta}{L} \propto \frac{1}{A}$ since k is constant. This implies that the smaller the cross-section area the bigger the temperature gradient value thus measurable.

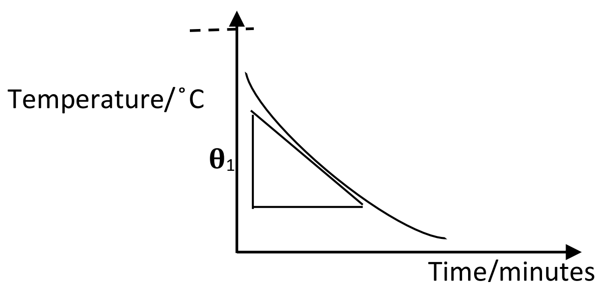
Poor conductor ("Lee's method")

- **Procedure.**

A <u>thin</u> disc S (specimen) of known <u>uniform</u> cross section area **A** and thickness **L** is sandwiched between steam chest **C** and a thick brass plate **B** of known heat capacity "**C_B**" as shown below.



The specimen S is heated by steam and at the steady state; values of θ_2 and θ_1 are read and recorded such that the rate of heat flow through S is $kA\frac{(\theta_2-\theta_1)}{L}$ ……(i)
The specimen S is removed and B is heated directly until its temperature is relatively higher than θ_1.
Steam chest is removed and B is lagged. Temperature of B is measured and recorded every after half a minute and its cooling curve is plotted as shown below.



The slope S_{θ_1} at temperature θ_1 is obtained such that the rate of heat loss by B is given as mcS_{θ_1}…..(ii) . And at steady state; the body looses heat at the same rate as it absorbs heat.

The rate of heat loss by B at $\boldsymbol{\theta}_1$ = The rate of heat B absorbed from S at the same temperature $\boldsymbol{\theta}_1$
$\Rightarrow$ $C_B S_{\theta_1} = kA\frac{(\theta_2-\theta_1)}{L}$ from which $k = \frac{C_B L S_{\theta_1}}{A(\theta_2-\theta_1)}$.

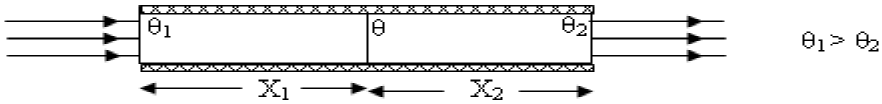
NB:
- Examples of poor conductors of heat are; glass, cardboard, wood, e.t.c.
- Searle's method is not appropriate because it is difficult to obtain adequate heat flow along the specimen (measurable temperature gradient).
- The adjoining faces of steam chest C, disc (specimen) and brass plate are flat and smeared with grease so as to ensure good thermal contact.

Precautions
(i) The upper and lower surfaces of the sample should be smeared with petroleum jerry to give a good thermal contact.

Series and parallel heat flow

(a) Series heat flow

If we consider two or more series sided slabs of thickness X_1 and X_2 with conductivities k_1 and k_2 in thermal contact and perfectly insulated as shown below



The rate of heat flow $\frac{dQ}{dt} = k_1 A \frac{(\theta_1 - \theta)}{x_1} = k_2 A \frac{(\theta - \theta_2)}{x_2} =$ Constant

NB: For series heat flow $R_e = R_1 + R_2 + ... + R_n$

Where R is called thermal resistance i.e. **thermal resistance** is the opposition to the flow of heat through a conductor of heat. Thermal resistance

R $= \frac{Thickness\ of\ a\ conductor}{Thermal\ conductivity x Area\ of\ transmission\ of\ the\ conductor}$ and the units for R are KW^{-1}.

NB:

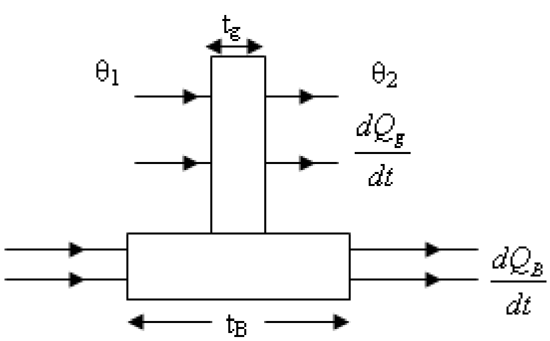
Double gazed windows are commonly used in cold countries because; such windows contain a thin layer of air. Air is poor conductor of heat since its conduction involves no material medium. There fore double gazed windows are in position to heat exchanges across it so as to maintain temperature inside the building at relatively room temperature.

Tea cups are made of clay other than alluminium because; clay is a poor conductor of heat yet aluminium is a good conductor of heat. Tea in clay cups take relatively longer time to cool than tea in an alluminium cup. A person taking tea using clay cup is least likely to be burnt as it may be the case if one takes tea using an alluminium cup.

(b) Parallel heat flow

This is common with heat flow through a vertical composite of different layers like;

- Brick wall with glasses
- Glass window composed of portions of different materials, e.t.c.



N.B: For parallel heat flow, temperature difference is the same, unlike in series heat flow.

The total rate of heat flow through the composite is;

$$\frac{dQ}{dt}=\frac{dQ_g}{dt}+\frac{dQ_B}{dt}$$

$$\frac{k_eA_e}{t_e}(\theta_1-\theta_2)=\frac{k_gA_g}{t_g}(\theta_1-\theta_2)+\frac{k_BA_B}{t_B}(\theta_1-\theta_2)$$

$\frac{1}{R_e}=\frac{1}{R_g}+\frac{1}{R_B}$, where;

R_e: Effective thermal resistance.
R_g: Thermal resistance of glass.
R_B: Thermal resistance of brick.

Examples

1) A metal boiler is 1.5cm thick. Find the difference in temperature between its inner and outer surface if every $1m^2$ of the boiler evaporates 40kg per hour (latent heat of vaporization of water=$2268\times10^3JKg^{-1}$, $m_{etal}=63Wm^{-1}K^{-1}$)

Solutions:

Rate of evaporation every $1m^2=\frac{dm}{dt}L_V$

Total rate of evaporation for area A $=\frac{dm}{dt}AL_V$

This heat causing evaporation = heat being supplied by the boiler$\left(\frac{kA\Delta\theta}{L}\right)$

$\Rightarrow\frac{k}{L}A\Delta\theta==\frac{dm}{dt}AL_V$ $\left(\frac{k\Delta\theta}{L}\right)=\frac{dm}{dt}L_v\Rightarrow\Delta\boldsymbol{\theta}=\frac{40x2268x10^3 0.15}{60x60x63}=60℃$.

2) A brick wall of thickness 0.12m and area $20cm^2$ has glass windows in it of area $2cm^2$ and thickness 0.4mm. The outside and inside surfaces of the wall have a temperature difference of 08K ($k_{brick}=7.0\times10^1WK^{-1}m^{-1}$, $k_{glass}=0.8WK^{-1}m^{-1}$). Find the effective rate of heat flow through the composition.

Solutions:

Method I

For parallel arrangement $\frac{dQ_T}{dt}=\frac{dQ_g}{dt}+\frac{dQ_B}{dt}$

$$=\frac{k_1A_1\Delta\theta}{x_1}+\frac{k_2A_2\Delta\theta}{x_2}$$

$$=\left(\frac{70*20*10^{-4}}{0.12}+\frac{0.8*2*10^{-4}}{0.4*10^{-3}}\right)*8=12.5W$$

Method II

Rate of heat flow $=\frac{\Delta\theta}{R_e}$

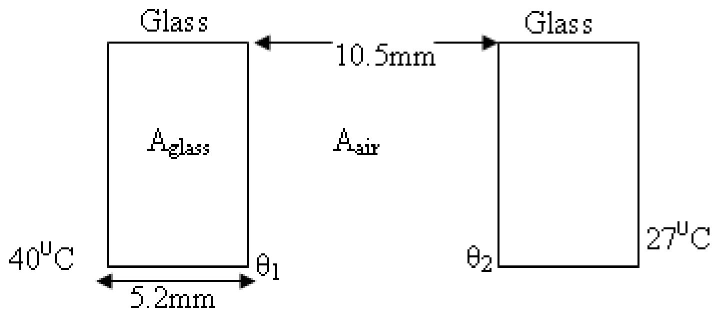
For a parallel array, $\frac{1}{R_e}=\frac{1}{R_g}+\frac{1}{R_B}$, $R_g=\frac{x_g}{k_g A_g}$ and $R_B=\frac{x_B}{k_B A_B}$

3) A double gazed glass window of area 3.5m^2 and thickness 5.2mm with a layer of air 10.5mm thick and the outside surface of the window is at 40^{o}C yet the inner surface is at 27^{o}C. Find

i) the rate of heat flow by conduction (k_{glass}=0.80Wm^{-1}K^{-1}, k_{air}=0.05 Wm^{-1}K^{-1})

ii) the temperature at the inner surfaces of the glasses.

Solutions:



Since air is trapped between the glass of area A_g, then $A_{air}=A_{glass}$

Method I

i) Rate of heat flow $=\frac{\Delta\theta}{R_e}$ for series heat flow

$$R_e=R_g+R_{air}+R_g=2R_g+R_{air}=\frac{2x_g}{k_g A}+\frac{x_{air}}{k_{air}A}$$

$$R_e=\frac{2*5.2*10^{-3}}{0.8*3.5}+\frac{10.5*10^{-3}}{0.05*3.5}=0.064KW^{-1}$$

Rate of heat flow $=\frac{40-27}{0.064}=204W$

ii)

From $\frac{dQ}{dt}=\frac{k_g A\Delta\theta}{x_g}$

$$\Rightarrow 204=\frac{0.80*3.5}{5.2*10^{-3}}\left(40-\theta_1\right)$$

$$\therefore \theta_1=39.6^o C$$

But for series heat flow $\frac{dQ}{dt} = \text{Constant}$

i.e. $204 = \frac{0.80 * 3.5 * 10^3}{5.2}(\theta_2 - 27)$

$\therefore \theta_2 = 27.38^o C$

Method II

Using $\frac{dQ}{dt} = \text{Constant}$

$$k_g A \frac{(40 - \theta_1)}{x_g} = k_{air} A \frac{(\theta_1 - \theta_2)}{x_{air}} = k_g A \frac{(\theta_2 - 27)}{x_g}$$

$$\frac{0.8(40 - \theta_1)}{5.2} = \frac{0.05(\theta_1 - \theta_2)}{10.5} = \frac{0.8(\theta_2 - 27)}{5.2}$$

$\Rightarrow$ For (i)=(ii)

$\theta_1 + \theta_2 = 67^o C$ ……………………. (a)

For (i) = (iii)

$0.15(40 - \theta_1) = 0.0048(\theta_1 - \theta_2)$

$31.25(40 - \theta_1) = \theta_1 - \theta_2$ ……………………… (b)

$31.25(40 - \theta_1) = \theta_1 - (67 - \theta_1)$

$\therefore \theta_1 = 39.6^o C$

$\theta_2 = 67 - 39.6 = 27.4^o C$

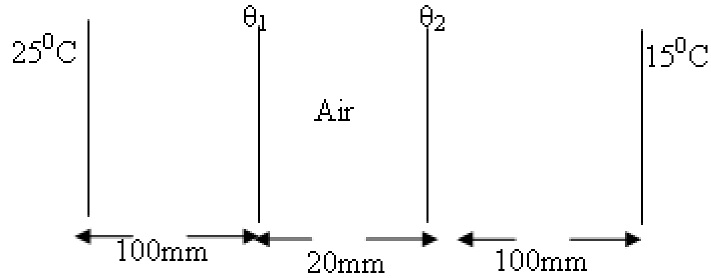
The rate of heat flow $\frac{0.8 * 3.5(40 - 39.6)}{5.2 * 10^{-3}} = 204W$

NB: It should be noted that the first method for calculating the rate of heat flow does not need us to know the inner temperature of the glass surfaces.

4. Ice is forming on the surface of water in a swimming pool. When it is 5.0cm thick, the temperature of the surface of the ice in contact with the air is 260K while the surface in contact with the water is at 273K. calculate:

i) the rate of heat loss per m^2 from the water. (598)
ii) rate at which the thickness of ice is increasing. Take thermal conductivity of ice = $2.3Wm^{-1}K^{-1}$, density of water = $1000kgm^{-3}$, specific latent heat of fussion = $3.25x10^5Jkg^{-1}$. ($1.86x10^{-6}ms^{-1}$)

5) A wall of a building consists of two brick layers each 100mm thick separated by 20mm thickness of air as shown below.



a) Calculate the rate of heat flow per unit area through the brick and air (k_{brick}=6.7Wm^{-1}K^{-1}, k_{air} is 0.024 Wm^{-1}K^{-1}).
b) Find the values of θ_1 and θ_2.

Solutions:

a) Heat flow per unit area $= \frac{\Delta\theta}{R_e^1}$ where

$R_e^1 = 2R_B^1 + R_{air}^1 =$ Thermal resistance x Area.

$$\therefore R_e^1 = 2\left(\frac{0.1}{6.7}\right) + \left(\frac{0.02}{0.024}\right) = 0.86KW^{-1}m^2$$

$$\Rightarrow \frac{\Delta\theta}{R_e^1} = \frac{25-15}{0.86} = 11.6Wm^{-2}$$

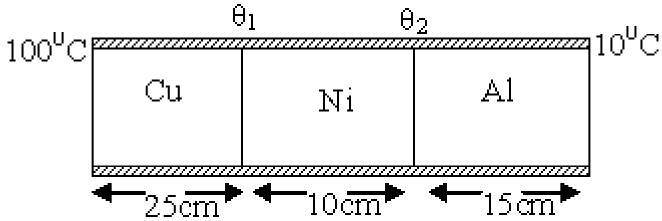
b) For the values of θ_1 and θ_2 we have

$$11.6 = 6.7 * \frac{(25-\theta_1)}{0.1} \Rightarrow \theta_1 = 24.8^o C$$

i.e. 25-0.17=θ_1 and θ_2=15+0.17=15.17^{o}C

5) A composite metal bar of uniform cross section is made up of length 25cm copper, 10cm nickel and 15cm Aluminum each being in perfect thermal contact with the ad joint part. The copper end of the composite bare is maintained at 100^{o}C and the Aluminum end is at 10^{o}C. The bar is perfectly insulated. Find the steady temperature at the junctions, and the rate of heat flow per unit area.

Solution



Rate of heat flow $=\frac{\Delta\theta}{dt}$

$R_e^1=\frac{x_{Cu}}{k_{Cu}}+\frac{x_{Ni}}{k_{Ni}}+\frac{x_{Al}}{k_{Al}}$

For (k_{Cu}=385Wm^{-1}K^{-1},k_{Ni}=59 Wm^{-1}K^{-1}and k_{Al}=209 Wm^{-1}K^{-1})

$R_e^1=\frac{0.25}{385}+\frac{0.10}{59}+\frac{0.15}{209}=0.00306\text{KW}^{-1}m^2$

Rate of heat flow per unit area $=\frac{90}{0.00306}=293928Wm^{-2}$

Using $\frac{dQ}{dt}=\text{Constant}$

$293928=\frac{385(100-\theta_1)}{0.25}$

$\theta_1=100-19.1=80.9^oC$

$\theta_2=10+19.1=29.1^oC$

6) A blackened copper sphere of mass 10kg, area 20m^2 and s.h.c 390Jkg^{-1}K^{-1} is cooled inside an evacuated enclosure whose walls are kept at 10°C. How long does it take for the sphere to cool from 200°C to 180°C? (Take k=14Wm^{-2}K^{-1})

Solutions:

Since a sphere has no thickness then

$\frac{dQ}{dt}=kA(\theta-\theta_o)$, where θ=sphere's temperature and θ_o is the surrounding temperature.

But also $\frac{dQ}{dt}=kA(\theta-\theta_o)=mc\frac{d\theta}{dt}$

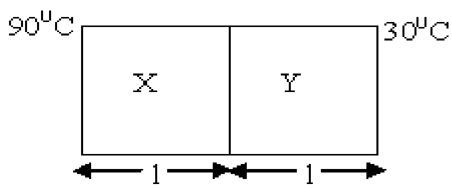
$\frac{d\theta}{dt}=\frac{kA}{mc}(\theta-\theta_o)=\frac{14*20*(200-0)}{10*390}=14.4Ks^{-1}$

$\Rightarrow$ Time taken for the sphere to cool from 200^{o}C to 180^{o}C $= \frac{200-180}{14.4} = 1.4s$

7) Two perfectly lagged identical metal bars X and Y are arranged in (a) series and (b) parallel. When in series, the hot end X is at 90^{o}C and the cold end Y is at 30^{o}C. When in parallel, the hot end of each bar is at 90^{o}C and the cold end of each bar is at 30^{o}C. Find the ratio of total rate of heat flow in the parallel to that of the series arrangements. Take thickness to be ***l***, cross section area **Am**^{2} and k_x=300Wm^{-1}K^{-1}, k_y==150Wm^{-1}K^{-1}.

Solutions:

a) Series arrangement

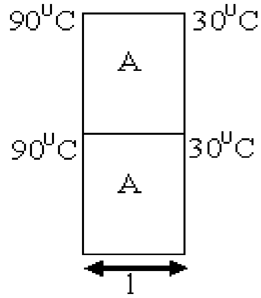


$$R_e = \frac{l}{300A} + \frac{l}{150A} = \frac{l}{100A}$$

$$\frac{dQ}{dt} = \frac{\Delta\theta}{R_e}$$

$$\Rightarrow \frac{dQ}{dt} = \frac{100A\Delta\theta}{l} = \frac{6000A}{l}W \text{ …………… (i)}$$

b) Parallel arrangement



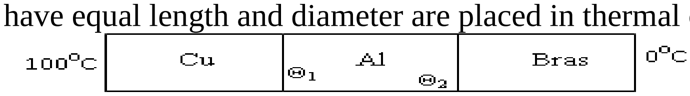
$$\frac{dQ^1}{dt} = \frac{\Delta\theta}{R_e^1}$$

$$R_e^1 = \frac{R_A * R_B}{R_A + R_B} = \frac{l^2}{45000A^2} \times \frac{100A}{l} = \frac{l}{450A}$$

$\therefore \frac{dQ^1}{dt} = \frac{60 * 450A}{l} = \frac{27000A}{l} W$ ………………… (ii)

Therefore, the ratio $\frac{dQ^1}{dt} : \frac{dQ}{dt} = 27 : 6 = 9 : 2$ or $\frac{dQ^1}{dt} \Big/ \frac{dQ}{dt} = 4.5$

8) A composite rod is made from uniform rods of copper, brass and aluminium. The rods which have equal length and diameter are placed in thermal contact and lagged as shown below.



Given that k_{Cu}:k_{Al};k_{Brass} is 18:10:5, determine the steady temperature at the copper-aluminum and aluminum-brass (Hint: assume uniform flow of heat).

Solutions:

$\frac{dQ}{dt} = k_{Cu}(100 - \theta_1) = k_{Al}(\theta_1 - \theta_2) = k_{Brass}(\theta_2 - 0)$

For $k_{Cu} : k_{Al} : k_{Brass} = 18 : 10 : 5$

$\Rightarrow 5\theta_2 = 1800 - 18\theta_1$ ……………………………. (i)

$100\theta_1 - 100\theta_2 = 1800 - 18\theta_1$ ……………………. (ii)

Equation (ii) – (i)

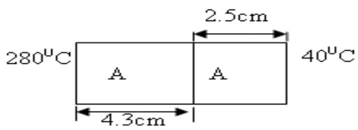
$10\theta_1 = 15\theta_2$, $\theta_1 = 1.5\theta_2$ Substitute into (i)

$\Rightarrow 5\theta_2 = 1800 - 27\theta_2$

$\theta_2 = 56.25^o C$ and $\theta_1 = 84.375^o C$

9).A copper rod 4.3cm thick and diameter 3.4cm is lagged. One end is maintained at 280^{o}C and the other end is placed against a disc of 2.5cm thick of the same diameter. The free end of the disc is maintained at 40^{o}C. If k_{Cu}=380Wm^{-1}K^{-1} and k_{disk}=10.5 Wm^{-1}K^{-1}, find the steady temperature of the copper-disc interface and the amount of heat flowing across the junctions in 10 minutes.

Solutions:



Rate of heat flow $= \dfrac{\Delta\theta}{R_e}$

$$R_e = \frac{4.3*10^{-2}}{380*\pi\left(3.4*10^{-2}\right)^2} + \frac{2.5*10^{-2}}{10.5*\pi\left(3.4*10^{-2}\right)^2} = 0.031 + 0.656 = 0.687KW^{-1}$$

Rate of heat flow $= \dfrac{240}{0.687} = 349.3W$

$\therefore$ The amount of heat flowing in 10 minutes $= 349.3*10*60 = 20960698 \approx 209607J$

$$\therefore 349.3 = \frac{380\pi\left(3.4*10^{-2}\right)^2}{4.3*10^{-2}}\left(280 - \theta_1\right)$$

$$10.88 = 280 - \theta_1 \Rightarrow \theta_1 = 269.12^0 C$$

10) Rods of copper, brass and steel are welded together to form a Y shaped figure. The cross sectional area of each rod is 2cm^2. The end of the copper rod is maintained at 100^{0}C and the ends of brass and steel at 10^{0}C. Assuming no heat loss from the surrounding from the surfaces of the rods, taking the lengths of the rods are 46cm, 20cm and 14cm, find the temperature at the junction and heat current (rate of heat flow) through the copper rod (k_{Cu}=385Wm^{-1}K^{-1}, k_{Brass}=109 Wm^{-1}K^{-1} and k_{steel}=50.2 Wm^{-1}K^{-1}).

Solutions:

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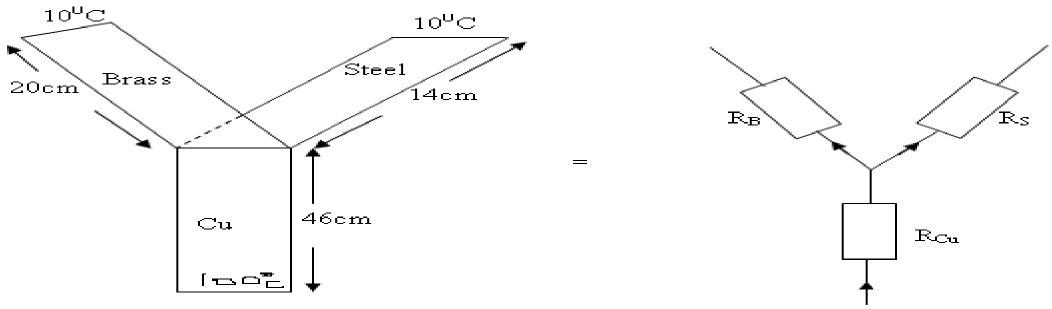


For uniform flow of heat, the rate of flow $=\dfrac{\Delta\theta}{R_e}$

$$R_e = R_e^1 + R_{Cu}\,,\ \ R_e^1 = \frac{R_B * R_s}{R_B + R_s} = \frac{\left(\dfrac{20*10^{-2}}{109*2*10^{-4}}\right)*\left(\dfrac{14*10^{-2}}{50.2*2.0*10^{-4}}\right)}{(9.174)+(13.944)} = 5.53KW^{-1}$$

$$R_e = 5.53+\left(\frac{46*10^{-2}}{385*2*10^{-4}}\right) = 11.504KW^{-1}$$

$\therefore$ The rate of heat flow through the copper rod $=\dfrac{100-10}{11.504} = 7.82W$

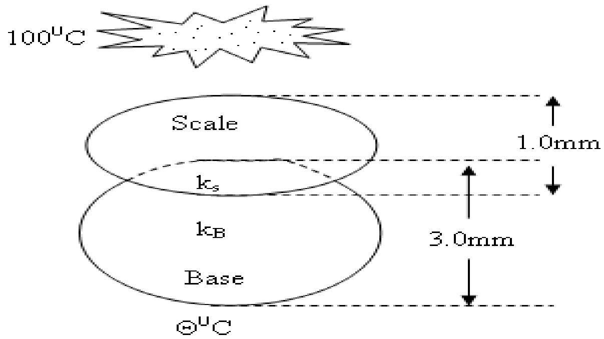
Also using $\dfrac{dQ}{dt} = \text{Constant}$

$$7.82 = \frac{385*2*10^{-4}}{46*10^{-2}}(100-\theta)$$

$$46.72 = 100-\theta \Rightarrow \theta = 53.28^0 C$$

11) A copper kettle has a circular base of radius 10cm and thickness 3.0mm. The upper surface of the base is covered with a uniform layer of scale 1mm thick. The kettle contains water which is brought to its boiling point by an electric heater and at steady state 5g of steam are produced each minute. What is the temperature of the lower surface of the base assuming that the heat conduction from the sides of the kettle is negligible. (K_{Cu} = 390WK^{-1}m^{-1},K_{scale}= 13.0WK^{-1}m^{-1} specific heat of steam = 2.26×10^{6} Jkg^{-1})

Solutions:



The rate of heat flow $=\frac{\Delta\theta}{R_e}$ $R_e = R_B + R_S$

$$R_e = \frac{3*10^{-3}}{390\pi\left(10*10^{-2}\right)^2} + \frac{1*10^{-3}}{13\pi\left(10*10^{-2}\right)^2} = 2.45*10^{-4} + 2.45*10^{-3} = 2.695KW^{-1}$$

The rate of heat flow $=\frac{(\theta-100)*10^{-3}}{2.695}$ …………………………….. (i)

This heat is the one which evaporates the water i.e.

$$\frac{(\theta-100)*1000}{2.695} = \frac{dM}{dt}*L_v = \frac{5*10^{-3}}{1*60}*2.26*10^6$$

$$\frac{(\theta-100)}{2.695} = \frac{5*2.26}{60} \Rightarrow \theta = 100.5/^0 C$$

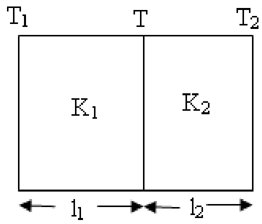
12 (a) A wall consists of two layers of thickness l_1 and l_2 and k_1 and k_2 respectively in series arrangement. If the surfaces of the walls are maintained in temperature T_1 and T_2, show that the rate of heat transfer is

$\frac{A(T_2-T_1)}{\left(\frac{l_1}{k_1}+\frac{l_2}{k_2}\right)}$, A is cross section area and $T_2>T_1$.

And state the assumptions based on.

(b) A cooking utensil of thickness 3mm is to be made from two layers i.e. Alluminium and brass. Determine which combination will allow high rate of heat flow by making use of the above expression (k_{Al}=240$Wm^{-1}K^{-1}$, k_{brass}=112 $Wm^{-1}K^{-1}$).

Solutions:



Assuming a steady state then $\frac{k_1}{l_1}(T - T_1) = \frac{k_2}{l_2}(T_2 - T)$

$$k_1 l_2 T - k_1 l_2 T_1 = k_2 l_1 T_2 - k_2 l_1 T$$

$$T(k_1 l_2 + k_2 l_1) = k_2 l_1 T_2 + k_1 l_2 T_1$$

$$T = \frac{k_2 l_1 T_2 + k_1 l_2 T_1}{k_1 l_2 + k_2 l_1}$$

$$\therefore \frac{dQ}{dt} = \frac{k_1 A}{l_1}\left(\frac{k_2 l_1 T_2 + k_1 l_2 T_1}{k_1 l_2 + k_2 l_1} - T_1\right)$$

$$= \frac{k_1 A}{l_1}\frac{(k_2 l_1 T_2 - k_2 l_1 T_1)}{k_1 l_2 + k_2 l_1}$$

$$= \frac{k_1 k_2 A(T_2 - T_1)}{k_1 l_2 + k_2 l_1}$$

$$= \frac{A(T_2 - T_1)}{\frac{l_2}{k_2} + \frac{l_1}{k_1}}$$

$$\Rightarrow \frac{dQ}{dt} \propto \frac{1}{\frac{l_2}{k_2} + \frac{l_1}{k_1}}$$

Therefore, for Alluminium of 2mm and brass of 1mm,

$$\frac{dQ}{dt} = \frac{k}{\frac{2*10^{-3}}{240} + \frac{1*10^{-3}}{112}} = \frac{k*10^5}{1.726}$$

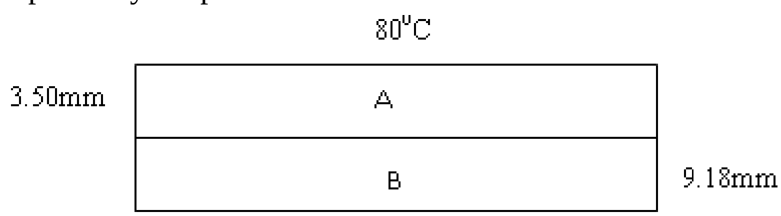
For Al=1mm and brass=2mm

$$\frac{dQ}{dt}=\frac{k}{\frac{1*10^{-3}}{240}+\frac{2*10^{-3}}{112}}=\frac{k*10^{5}}{2.202}$$

Since 2.202>1.726, then $\frac{dQ}{dt}$ is higher for Al=2mm and brass=1mm.

8.0.1.4Test8

1) Two metal conductors A and B each of radius 18cm and thickness 3.5mm and 9.18mm respectively are placed in contact with their base at 180℃ and the upper at 80℃ as shown below.



a) Given that k_A=120Wm^{-1}K^{-1}, k_B=155 Wm^{-1}K^{-1}, find the temperature of the interface and the rate of heat flow through B.

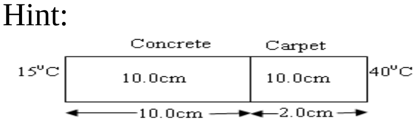
b) Given that metal B is disjoined from A such that B is placed in an environment at a temperature of 25^{o}C. One end of it is heated at a temperature of 220^{o}C, find how long it will take for the other end to be raised to 170^{o}C if m_B=2.4×10^{-3}kg and s.h.c of B is 810Jkg^{-1}K^{-1}.

2) A window of height 2.0m and width 0.8m is double gazed consisting of single glass plates each of thickness 4.0mm separated by an air gap of 2.8mm. Find the rate at which heat is conducted through the window given that the external surfaces temperatures are 20^{0}C and 30^{0}C (k_g=0.72WK^{-1}m^{-1}, k_{air}=0.025 WK^{-1}m^{-1}).

3) A concrete floor of a hall has dimensions of 10.0m by 8.0m. It is covered with a carpet of thickness 2.0cm. Temperature inside the hall is 40^{0}C while that of the surrounding just below the concrete is 15^{0}C. Thermal conductivities of the concrete and the material of the carpet are 2.4Wm^{-1}K^{-1} and 0.05 Wm^{-1}K^{-1} respectively and the thickness of the concrete is 10cm. Find

i) temperature at the interface of the concrete and carpet
ii) the rate of heat flow through the floor

Hint:



4)Water in an aluminum saucepan of diameter 16cm and thickness 4mm is kept boiling at 100^oc on a hot stove. The water boils of at a rate of $2.28\times10^{-4}kgs^{-1}$. Find the temperature of the underside of the saucepan, assuming it is uniformly heated and neglecting heat losses from the sides. ($K_{Al} = 2.06\times10^2\ WK^{-1}m^{-1}$, s.l.h.v of water = $2.26\times10^6 Jkg^{-1}$)

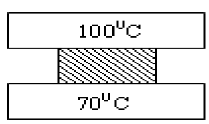
5) The bottom of a cylindrical metal can have a diameter of 10.0cm and thickness 1.5cm. Water in the can is kept 100℃ over a frame. If 0.8kg of water evaporates in 1 hour, find the average temperature of the outer surface of the can. ($K_{metal} = 386\ WK^{-1}m^{-1}$, Latent heat of vaporization of water $2.26\times10^6 Jkg^{-1}$).

6) Four metal rods copper, steel, brass, and aluminum are welded to form a heater in the shape of letter "**E**". Copper rod is of area $8m^2$ and thickness 2cm. steel brass and aluminum rods are each of area $2m^2$ and thickness 6cm. the free end of copper is maintained at 120^oC. and those of the other three rods are maintained at 05^oC.

a) By neglecting heat loss to the surrounding, find the temperature of the copper-aluminum interface and rate of heat flow through the brass rod.
b) Repeat (a) above when
i) Steel rod is disordered from the above heater in No.6 above
ii) Brass rod is disordered from the above heater in No.6 above

($K_{cu} = 385\ WK^{-1}m^{-1}$, $K_{steel} = 50.5\ WK^{-1}m^{-1}$, $K_{brass} = 109\ WK^{-1}m^{-1}$, $K_{Al} = 206\ WK^{-1}m^{-1}$)

4) In an experiment to determine the thermal conductivity of glass, sheets of glass were cut to form a circular disc of 5cm radius and had a uniform thickness of 2mm. One side was maintained at a steady temperature of 100^oC, while a copper block in good thermal contact with the glass was found to be at 70^oC. The block weighed 0.75kg. Following this, the rate of cooling of the copper block was studied over a range of temperatures and the rate of cooling at 70^oC was found to be $16.5Kmin^{-1}$. Find the value for the thermal conductivity of glass. (s.h.c of Cu=$400Jkg^{-1}K^{-1}$)



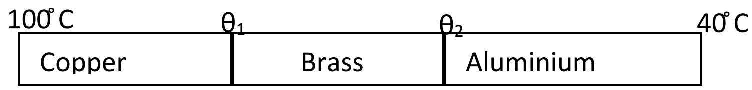
$$k = \frac{ms_\theta cl}{A(\theta_2 - \theta_1)}$$

8) A window of height 1m and width 1.5m contains a double gazed unit consisting of two single glass planes of thickness 4mm separated by an air gap of 2mm. Find the rate at which heat is flowing through the window. Take the temperatures of the external surfaces to be 20^oC and 37^oC, $k_g=0.72WK^{-1}m^{-1}$, $k_{air}=0.025WK^{-1}m^{-1}$.

9) A lagged copper rod 2m long and 3cm in diameter has one end maintained at 85°C. The other end is placed against a 2cm thick cardboard disc of the same diameter as the rod. The free end of the disc is maintained at 40°C (k_{Cu}=380Wm^{-1}K^{-1}, $k_{cardboard}$=0.2Wm^{-1}K^{-1}). Find the interface temperature and the quantity of heat which flows in 10minutes.

10) (a) Define thermal conductivity and temperature gradient

(b) (i) Copper, brass and Alluminium rods of length 10.0cm, 5.0cm and 10.0cm respectively are joined as shown below



Thermal conductivities of these metals are in the ratio of 20:18:15 respectively find θ_1 and θ_2

(ii)If their cross section area is 2cm^2, find the amount of heat which would flow through Brass after 1hour.

(c) Describe the experiment for determining thermal conductivity of glass.

11) (a) (i) What is meant by **conductionconvection** and **radiation** as modes of heat transfer.

(ii) Explain why double –gazed windows are commonly used in cold countries.

(iii) Explain why tea cups are made of clay other than aluminium.

(b) Show how temperature is distributed along the length of;

(i) Lagged conductor

(ii) Unlagged conductor

Whose ends are maintained at θ_1 and θ_2 where $\theta_1 > \theta_2$

c) Account for the graphs sketched in 11(b) above

d) A rectangular concrete floor of a hall has dimensions of 10.0m by 8.0m. It is covered with a carpet of thickness 2.0cm. The inside the hall is at 40°C while that of the surrounding just below the concrete is at 15°C. $K_{concrete}$ =2.4Wm^{-1}k^{-1}. K_{carpet} = 0.05Wm^{-1}k^{-1} and thickness of concrete is 10.0cm. Find the;

(i) Temperature at the interface of concrete and carpet.

(ii) Rate of heat flow through the floor.

e) Explain four precautions taken when determining thermal conductivity of a metal.

12) The ground floor of a house has an internal area of 5.0m^2. The floor is fitted with a carpet 15mm thick which completely covers the floor of thickness 200mm. The top of the carpet is at 15℃ and the bottom of the concrete is at 10℃ (k_{floor}=0.75Wm-1K^{-1}, and k_{carpet}=0.06Wm^{-1}K^{-1}). Calculate;

(i) Rate of energy transfer through the concrete with out the carpet.

(ii) Temperature at the carpet/concrete interface.

13) In an experiment to determine thermal conductivity of copper, heat is supplied at a rate of 80W to one end of a well lagged uniform copper bar of cross-section area 10.0cm^2 and total length 20cm. Heat is removed by water cooling at the other end of the bar. Two thermometers T_1

andT_2 are used to record the temperature of the bar at distances 5.0cm and 15.0cm from the hot end. At steady state these thermometers read 48℃ and 28℃ respectively (s.h.c of water = $4200Jkg^{-1}K^{-1}$). Calculate;

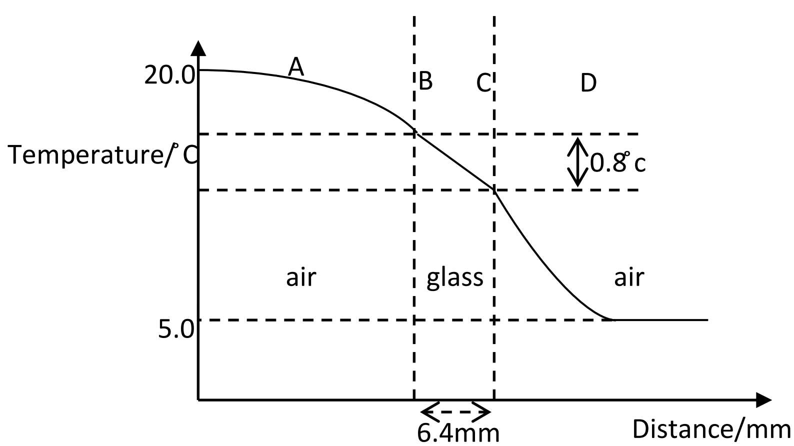
(i) Thermal conductivity of copper.

(ii) Rate of flow of water in (grams min^{-1}) of cooling water for its temperature rise to be 5K.

(iii) Temperature at the cold end of the bar.

14) In an experiment for determining thermal conductivity of cardboard, a piece of area $25.0cm^2$and thickness 2.00mm is cut from it to be used as a specimen. Heat from brass plate at 99.0℃ enters one end of this piece and passes to a copper block of mass 600g from which it escapes to the surrounding air. At steady state temperature of copper block is 59.0℃. The piece is removed and copper block is heated directly further to 60.6℃ and properly lagged and left to cool. The rate of temperature fall for this copper block at 59.0℃ is found to be $2.51Kmin^{-1}$ ($k_{cardboard}$=410Jkg-1K-1). Calculate thermal conductivity of the card board.

15) The diagram shows the variation temperature with distance for a glass window through which thermal energy is being lost at a rate of $900Wm^{-2}$.



(i) Explain the shape of this graph in regions AB and CD.

(ii) Calculate the temperature through glass.

(iii) Calculate the “U” value for this window.

(iv) Sketch the graph you would expect for a double gazed window assuming that temperature at the outer surfaces of the window are 20.0℃ and 5.0℃.

THERMAL RADIATION

Thermal radiation is the transfer of heat through vacuum i.e. no material medium is required for this transmission.

Electromagnetic spectrum is the distribution of electoral magnetic radiations ranging from those of short wave length to those of longer wave length as shown below.

Type of radiation	Wave length range (m)
Gamma rays	10^{-12}
X- rays	$>10^{-12}$ to10^{-8}
Ultra-violet	$>10^{-8}$ to 10^{-7}
Visible-violet	$>10^{-7}$ to $4.0x10^{-7}$
Visible-red	$>4.0x10^{-7}$ to $7.4x10^{-7}$
Infrared	$>7.0x10^{-7}$to 10^{-3}
Microwaves	$>10^{-3}$ to 10^{-1}
Radio waves	$>10^{-1}$ to10^{4}

V
I
B
G
Y
O
R

Properties of thrermal radiation

(i) They travel through vacuum with a speed of light.
(ii) They travel in straight lines.
(iii) They obey the laws of reflection and refraction of light.
(iv) They are diffracted, suffer interference and polarization. etc.

Infrared radiations

Infrared radiations are electromagnetic radiations which are converted into heat when they strike a surface. Infrared radiations are called so because they occur beyond the red end of the visible spectrum.

Question

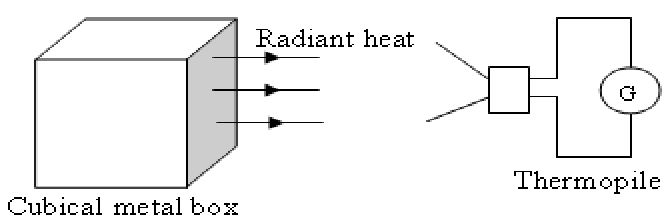
Explain why infrared radiations are called so.

- **Properties of infrared radiations.**

- They travel at the speed of light.
- They are reflected and refracted like light.
- When absorbed by a body, the body's temperature is raised.
- Causes photo-electric emission from surfaces like Cesium.
- Affects special types of photographic plates which enable pictures to be taken in the dark.
- They are absorbed by glass but transmitted by rock salt and quartz.

Comparison of radiation ability for different surfaces

A cubical metal tank whose sides are painted; dull black, dull white and highly polished is filled with hot water and radiations from each surface are detected by a thermopile as shown below.



The galvanometer deflection is greatest when the thermopile is facing the dull black surface and least when facing a highly polished silver surface.
Therefore, a polished surface is the least radiator and a black surface is the best radiator.

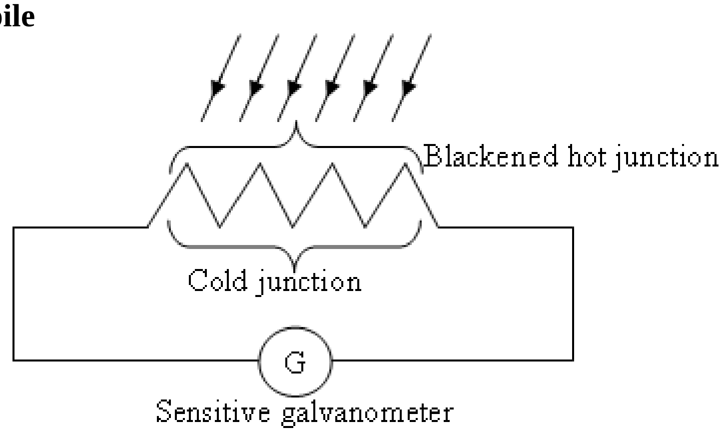
Explain why cloudy nights are warmer compared to cloudless nights.
During day, the Earthn absorbs heat from the sun and at night, it loses heat to the atmosphere. On cloudy nights, the heat radiated by the earth is reflected back by the clouds and the earth feels warm. On cloudless nights, the heat radiated by the Earth is lost into the atmosphere.

NB:

- People in desert countries are always dressed in white clothes other dark clothes because;
  White clothes are good reflectors of heat yet dark clothes are good absorbers of heat. There fore people in desert countries need to put on white clothes so as to reflect much of the heat incident on them such that less heat reaches their bodies.
- Many factory roofs are aluminum-painted because bright surfaces reduce heat lost by radiation during winter and enable the interior to remain cooler during summer.
- In sub-Sahara desert, it is by rule to put on white clothings for the same reason.

Detection of radiant heat (detection of infrared radiation)
There are mainly three types of detectors i.e.

(a) Thermopile



- Thermopiles are connected in series and to the sensitive galvanometer.

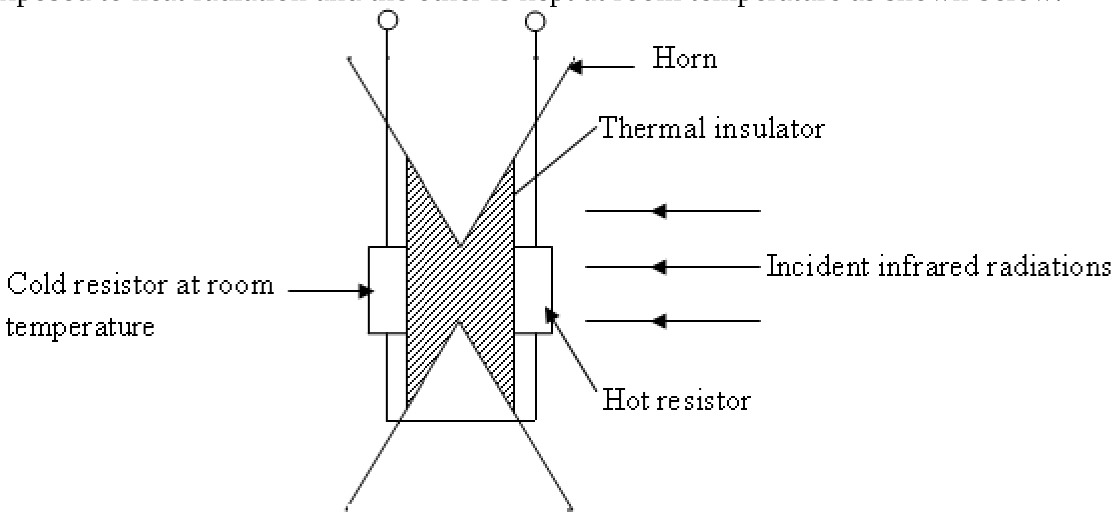
- When radiations are incident on the thermocouples, they warm up the junction and sets up an e.m.f which is proportional to the temperature difference between the hot and cold junctions.
- The galvanometer deflects and the magnitude of deflection is a measure of the intensity of the incident radiations on the thermopile.

NB_1:

- The hot junctions of thermopiles are conic and highly polished so as to concentrate the radiations on the junctions.
- The hot junctions are blackened to make them good absorbers of incident radiations.
- The galvanometer should be highly sensitive so as to be in position to detect radiations of relatively small intensity.

(b) Bolometer

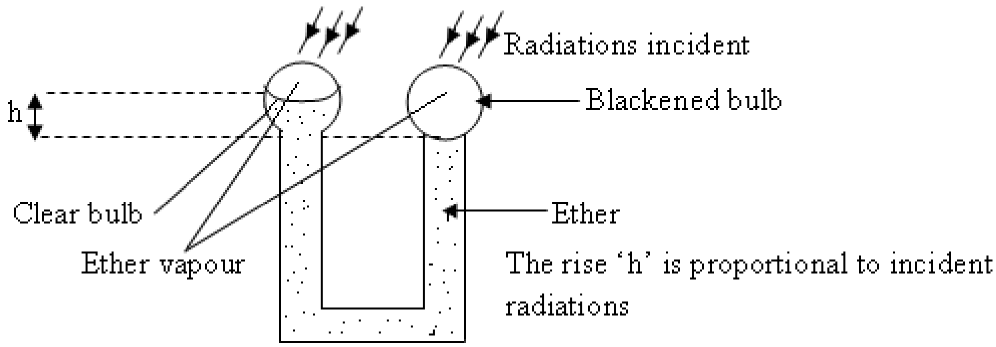
This consists of two heat sensitive resistors (thermostats). One of them is coated black and exposed to heat radiation and the other is kept at room temperature as shown below.



Radiations are absorbed by the hot resistor causing an increase in its resistance which is proportional to the intensity of the incident radiations and this increase in resistance is determined by the use of a wheastone bridge.

(c) **The ether-thermo scope**.

A mixture of air and ether vapour is trapped in a tube partly filled with liquid ether.
When infrared radiations fall on the apparatus, the liquid ether rises into the clear bulb while the level falls in the blackened bulb.
This is because the radiations that fall on the apparatus is absorbed more by the blackened bulb than a clear one.
This shows that a blackened surface is a better absorber of thermal radiation than a polished one.



Black body

A black body is body which absorbs all radiations incident on it and reflects or transmits none.

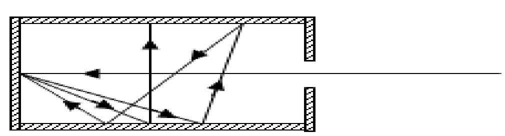
A black body radiation is an electromagnetic radiation emitted by a body solely due its temperature i.e. energy emitted depends on body's temperature or because the relative intensities of the various wave length present depend only on the temperature of then body.

Black body radiator is a body which emits radiations which are characteristics of its temperature and does not depend on the nature of its surface.

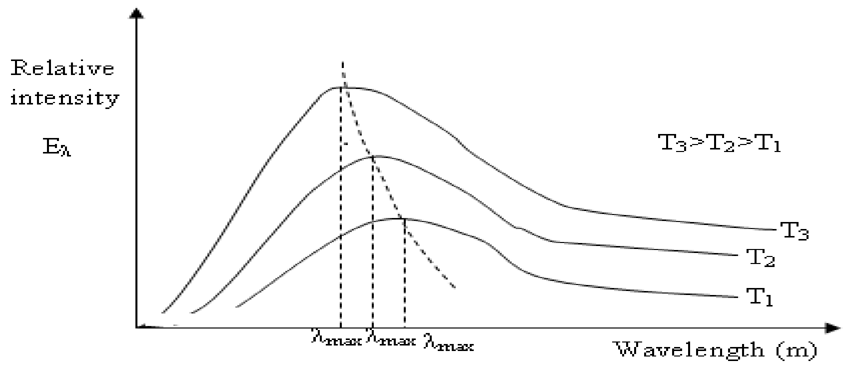
Note: The radiations emitted by a black body are called *temperature radiations or black body radiations because they depend on the temperature of the body*

Approximation of black body

- This is done by punching a small hole in an enclosure whose inside walls are rough and painted black.
- At each reflection inside the cavity, a high percentage of the radiations is absorbed and eventually all radiations are absorbed after multiple reflections as shown below.



Distribution of black body radiation



Description of the main features of the curve

- For all wave lengths, an increase in temperature causes an increase in energy of emission.
- The wavelength at which maximum intensity occur shifts to the shorter wavelength as temperature is increased.
- At a given temperature, the energy is not uniformly distributed in the radiation spectrum of a body ie energy for different wave lengths is different.
- At a given temperature, the energy of emission (relative intensity) has maximum value for a particular wave length λ_{max}.
- The area under each curve represents the total energy emitted for a complete spectrum at a particular temperature.

Change of colour in relation to black body radiation

- Relative intensity increases with increase in temperature and λ_{max} decreases as temperature is increased.
- The colour of the metal being heated depends on the position of λ_{max} in the visible spectrum (**ROYGBIV**) i.e. a body is red if λ_{max} is in the red region of visible spectrum, yellow if λ_{max} is in the yellow region of the visible spectrum or white when λ_{max} is in the middle of the visible spectrum and finally blue when λ_{max} is in the blue spectrum visible region.

NB:

The center of fire scene appear white because;

The scene of fire is hottest at the center. Since intensity of emitting black body is directly proportional to its temperature yet intensity increases rapidly for shorter wave lengths, At the center of fire scene, very many radiations whose wave lengths are in the visible spectrum are emitted. A mixture of these radiations of varying wave lengths constitutes the whiteness of the center of the fire scene.

Explain why welders should put on glasses.

A welder puts on dark glasses which absorb ultraviolet radiations which destroys the retina. This cuts down the intensity of light.

Laws of blackbody radiation

1. **Wien's displacement law** states that as the temperature increases, the maximum intensity of the emission shifts (is displaced) towards the shorter wave length.

Note:

The wavelength $\lambda_{\max}$ of radiations emitted by a black body at maximum intensity/emissions is given by $\lambda_{max}T$ = constant=0.002892mK where; T is the body's absolute temperature.

Example

1. Use the figure below to find $\lambda_{\max}$ which corresponds to curves with peaks for;

i) radiation in the sun's core where the temperature is 15×10^6K

ii) radiation in an interstellar space whose temperature $\approx$ 2.7K

(a) Name the radiations emitted in each case.

Solutions:

Using Wien's displacement law $\lambda_{\max}T = \text{Constant}$

i) At T=15×10^6K

$$\lambda_{\max} * 15 * 10^6 = 600 * 5800$$
$$\lambda_{\max} = 0.232nm$$

"X-ray radiation"

ii) At 2.7K

$$\lambda_{\max} * 2.7 = 600 * 5800$$
$$\lambda_{\max} = 1.3mm$$

"Infrared radiation"

Emissive power and emissivity

Different sources of heat radiation emit different amounts of energy even when they are at the same temperature. The amount of heat energy emitted (radiated) per second by a body depends upon (i) the area of its surface, (ii) the difference of temperature between surface and the surroundings, and (iii) the nature of its surface. The strength of emission is estimated by a quantity known as emissive power.

Definition. ***Emissive power*** is the amount of heat energy emitted per second from unit area of the radiating surface. It is denoted by e. it depends only on the nature of the surface and its temperature.

Emissivity (ε ***):*** is the ratio of its emissive power to that of a perfect radiator (black body) at the same temperature. i.e.

$$\boldsymbol{\varepsilon} = \left(\frac{\textbf{Total power radiated per square meter of any body}}{\textbf{Total power radiated per squared meter of a black body}}\right)_{T} \ldots\ldots \textbf{(ii)}$$

Equation (ii) above implies that;

Total power radiated per squared meter of any body P'_T=e x Total power radiated per squared meter of a black body……… (iii)

Combining equations (i) and (iii), we deduce that; total power radiated per squared meter of any body at temperature "T" is given by P'_T =eσT^4………. (iv)

Equation (iv) implies that all perfectly black bodies, their emissivity is "**1.0"**

NB: The ratio of total power emitted by a body at equilibrium temperature "T" to its effective surface which emits is called the body's "**Irradiance"** i.e.

Irradiance $= \frac{Total\ power\ radiated\ by\ a\ body\ at\ equilibrium\ temperature\ "T"}{Body's\ effective\ surface\ area\ of\ emission}$ =eσT^4 or σT^4 for perfectly black bodies.

2. **Stefan's law states** that the total energy emitted by a unit area of a perfect radiator per second is proportional to the fourth power of its absolute temperature.

If Q ($\frac{P_T}{A}\boldsymbol{\alpha}T^4$)is the total energy of all wave lengths emitted per unit area per second by a black body at absolute temperature T, then, $Q \propto T^4$, or $Q = \sigma T^4$ … … … … … … … … … .1

For a body of emissivity ε, we have $Q = \varepsilon\sigma T^4$………………………..2

Where σ is stefan's constant i.e. $\boldsymbol{\sigma = 5.67 \times 10^{-8} Wm^{-2}K^{-4}}$

Example.

1. Calculate the radiant energy per unit area per second of a black body at a temperature of 1200K given σ = $5.67\text{x}10^{-8}Wm^{-2}K^{-4}$. ($1.19\text{x}10^5$)
2. Calculate the energy radiated per second by a sphere (assumed to be a black body radiator) of diameter 10cm maintained ata constant temperature of 727^oC. givenσ = $5.67\text{x}10^{-8}Wm^{-2}K^{-4}$. ($1.8\text{x}10^3$W). hint surface area of a sphere is $4\pi r^2$.
3. A cylindrical bulb filament of length 0.5m and radius 1.0×10^{-4}m emits light as black body. 0.4A melts the filament when connected across 240V. Calculate;
   (i) The melting point of the filament
   (ii) The wave length of the radiation emitted at maximum intensity/emission at its melting point.

Solutions:

i) Area $A = 2\pi rl = 2\pi * 1.0 * 10^{-4} * 0.5 = \pi * 10^{-4} m^2$

Power $= IV = A\delta T^4$

$$\Rightarrow T = \left(\frac{240 * 0.4}{5.7 * 10^{-8} * \pi * 10^{-4}} \right)^{1/4} = 1522K$$

ii) For $\lambda_{\max} T = 2.9 * 10^{-3}$

$$\lambda_{\max} = \left(\frac{2.9 * 10^{-3}}{1522} \right) = 1.91 * 10^{-6} m \approx 1.91 \mu m$$

4. A tungsten filament lamp of 10W lamp at a temperature of 217°C and effective surface area of 0.64cm^2 radiates energy at a rate equivalent to 49% of that radiated by a black body. Calculate Stefan's constant.

Solutions:

Power of the lamp $= IV = \frac{49}{100} * \delta A T^4$

$$10 = 0.49 * \left(0.64 * 10^{-4}\right)^2 \delta (490)^4$$

$$\delta = \frac{40}{(0.49) * (490)^4 * \left(0.64 * 10^{-4}\right)}$$

$= 5.67 \times 10^{-8} Wm^{-2}K^{-4}$

5. a) 5.0A flows through a 4.0m long metal filament of an electric lamp of diameter 0.4mm and resistance per centimeter of 22Ωcm^{-1}. Calculate the filament temperature and frequency of the radiation emitted at maximum intensity/emission given that the body radiates as a black body of emissivity 1.0.

Solutions:

i) Assuming the filament to radiate like a blackbody then,

$$I^2 R = \delta A T^4$$

$$5^2 * 22 * 400 = 5.67 * 10^{-8} * 2\pi \left(0.2 * 10^{-3}\right) * 4T^4$$

$$\Rightarrow T = 5270.99K$$

ii) Using Wien's displacement law

$$\lambda_{\max} \left(1234 * 10^{11}\right)^{1/4} = 2.9 * 10^{-3}$$

$$\therefore f = \frac{5270.99 x 3.0 x 10^8}{\left(2.9 * 10^{-3}\right)} = 5.5 x 10^{14} Hz$$

6. A closed cube of side 1.0cm has a grey surface which radiates 50% as a black body at 700℃. Calculate;

(i) Power radiated by the cube

Solutions:

Power = $0.5 \times 5.67 \times 10^{-8} \times 6.0 \times 10^{-4} \times (973)^4 = 15.33W$

(ii) The radius of a spherical black body which would radiate the same power calculated in 10(i) above when the body it is at 300℃.

Solutions:

15.33 =5.67x10^{-8}x4xπxRx $(573)^4$

$$\mathbf{R} = \sqrt{\frac{\mathbf{15.33}}{\mathbf{5.67x10^{-8}x4x\pi x(573)^4}}} = \mathbf{0.014m}$$

7. A cylindrical electric element of length 25cm and diameter 1.5cm is rated 1000W. Determine the filament's equilibrium temperature if it is assumed to behave like a black body.

Solutions:

We use $IV = \delta AT^4$

$$1000 = 5.67*10^{-8} * 2\pi\left(0.75*10^{-2}\right)*\left(25*10^{-2}\right)*T^4$$

$$T = \left(1.49704826*10^{12}\right)^{1/4} = 1106.1K$$

Prevost's theory

States that when the temperature of a body is constant, then the body is loosing heat by radiation and gaining by absorption at equal rates.

A body is said to be at equilibrium temperature when its rate of absorption is equal to its rate of emission i.e. the total power leaving the body per squared meter must be equal to the total power falling on it.

Radiations inside a constant temperature enclosure

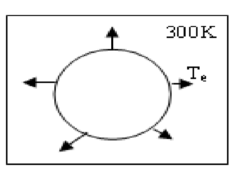
When a perfectly black body of surface area "**A**" at temperature T_1 is enclosed in an enclosure at higher temperature $T_2>T_1$, its net heat gain is given by P_g = **σ**A $(T_2^4 - T_1^4)$ and if the body's temperature was higher than that of the enclosure then its net heat loss to the enclosure is given by P_L =**σ**A $(T_1^4 - T_2^4)$.

Examples

1. A solid copper sphere of diameter 20mm is cooled to a temperature of 500K and is then placed in an enclosure maintained at 300K. Assuming that all the exchange of heat is caused by radiation, calculate the initial rate of temperature fall if the sphere is assumed to emit like a blackbody.

Solutions:

Assume $\rho_{Cu} = 8930Kgm^{-3}$; enclosure to have the same shape of the sphere, s.h.c of copper is 370J$Kg^{-1}K^{-1}$.



By principle; the net of heat loss by the body $= P_e - P_b = \delta A\left(T_e^4 - T_b^4\right)$

But by definition this rate of heat loss is given by $= mc\dfrac{\Delta\theta}{\Delta t}$

$$\therefore 5.67*10^{-8}*4\pi*\left(1.0*10^{-2}\right)^2\left(500^4-300^4\right)=\frac{4}{3}\pi\left(1.0*10^{-2}\right)^3*8930*370*\Delta\theta$$

$$\Rightarrow \frac{\Delta\theta}{\Delta t}=0.28Ks^{-1}$$

2. A metal sphere of diameter 1.0×10^{-2}m is cooled to a temperature of 250K and is then placed in an enclosure maintained at 400K. Assumed that all heat exchange is by radiation to calculate the initial rate of rise of temperature of the sphere, assume the blackbody to be similar to the sphere ($\rho_{metal} = 7.20*10^3 kgm^{-3}$, s.h.c of metal is 350$Jkg^{-1}K^{-1}$and $\delta = 5.70*10^{-8}Wm^{-2}K^{-4}$)

Solar constant

Solar constant ($S=\sigma T^4$) is the sun's radiant power received on a squared meter of the outside surface of the earth's atmosphere which is perpendicular to the direction of the incident radiations at the sun-earth mean distance. From $P = A\sigma T^4 \Longrightarrow S = \frac{P}{A} = \sigma T^4$.

Suppose the power radiated by the sun is $P_s = 4\pi r_s^2\sigma T_s^4$. If the Earth is at a distance R from the Sun, then solar power falls on the total surface area $4\pi R^2$. Solar power falling on a unit surface area is given by $\frac{P_s}{A} = \frac{4\pi r_s^2\sigma T_s^4}{4\pi R^2}$.

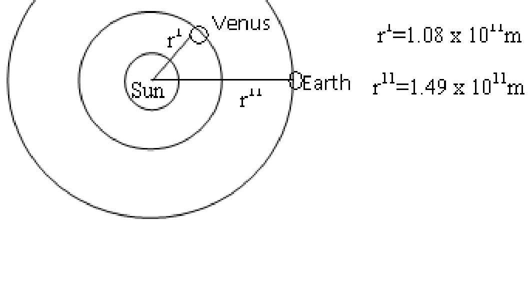
Example

1) The mean distance from the sun to the earth is 1.49×10^{11}m and from the sun to Venus is 1.08×10^{11}m. The absorption rate of the sun and the earth is the same and the earth's solar constant is $S=1.4KWm^{-2}$. Calculate;

i) The solar constant of the Venus.

ii) The equilibrium temperature of Venus assuming that it absorbs and radiates as a black

Solutions:



i) Power absorbed by the earth from the sun is $S * 4\pi r^{11^2}$

Power absorbed by the Venus is $S^1 * 4\pi r^{1^2}$

At equilibrium temperature the rate of absorption by the two planets are equal

$\Rightarrow S * 4\pi r^{11^2} = S^1 * 4\pi r^{1^2}$

$$\therefore S^1 = \frac{1.4 * 10^3 (1.49)^2}{(1.08)^2} = 2.66 * 10^3 Wm^{-2}$$

ii) By definition, solar constant $S^1 = \delta T_o^4$

$$T_r = \left(\frac{2.66 * 10^3}{5.67 * 10^{-8}}\right)^{1/4} = 465K$$

2. The solar constant which is the energy arriving per second at the Earth from the sun is about $1400Wm^{-2}$. Estimate the surface temperature of the Sun, given that the Sun's radius is $7x10^5$km, the distance of the sun from the Earth is $1.5x10^7$km and staefan's constant is $5.7x10^{-8}Wm^{-2}K^{-4}$.

Solution

$$\frac{P_S}{A} = \frac{4\pi r_S^2 \sigma T_S^4}{4\pi R^2} = 1400 \Rightarrow T_S^4 = \frac{1400}{\sigma}\left(\frac{R}{r}\right)^2 \Rightarrow T_S = 5795K$$

Surface temperature of the Earth

This can be estimated on the assumption that the Earth is in radiative equilibrium with the sun. This assumption means that,

Power radiated by the Earth = Power received by the Earth from the Sun

Power radiated by the sun $P_s = 4\pi r_s^2 \sigma T_s^4$

The geometrical projection of the sun on the Earth is a circle whose radius is the radius of the Earth, r_e. Thus the effective area on which the power falls at right angles is $\boldsymbol{\pi r_e^2}$. But the total area on the solar power falls is $\boldsymbol{4\pi R^2}$, where R is the distance of the Earth from the Sun. So the fraction of solar radiation incident on the Earth is given by $\boldsymbol{\frac{\pi r_e^2}{4\pi R^2}(4\pi r_s^2 \sigma T_s^4)}$

Power radioted by the Earth = $\boldsymbol{4\pi r_e^2 \sigma T_e^4}$

For radiative equilibrium, power radiatited by the Earth = power received by the Earth,

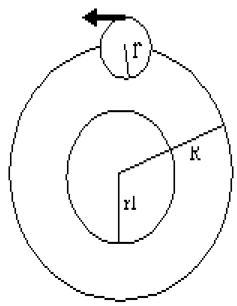
$$\boldsymbol{4\pi r_e^2 \sigma T_e^4 = \frac{\pi r_e^2}{4\pi R^2}(4\pi r_s^2 \sigma T_s^4) \Rightarrow T_e = T_s\sqrt{\frac{r_s}{2R}}}$$

Also; solar energy = $\frac{\pi r_e^2}{4\pi R^2}$ **x energy radiated by the Sun**

Example

1. Find the equilibrium temperature T_e of the planet earth which revolves around the sun in an elliptical orbit of radius 1.5×10^{11}m (Temperature of the sun temperature is T_s=5800K, Stefan's constant=$5.67\times10^{-8}Wm^{-2}K^{-4}$, sun's radius is 7.0×10^8m, emissivity of the sun and earth is 1.0 and earth's radius is $6.4x10^6$m)

Solution



By Stefan's law, the power radiated by the sun $= \delta A T^4$

$$P_{rad} = 5.67*10^{-8}*4\pi(7.0*10^8)^2*(5800)^4 = 4.0*10^{26}W$$

Assuming the area enclosed by the two planets to be spherical

Power transmitted by 1m^2 of this area $= \frac{P_{rad}}{4\pi R^2}$

Power absorbed by the earth from the sun $P_a = \frac{P_{rad}}{4\pi R^2}*A_{earth}$

Where A_{earth} is the effective area of the earth on which radiations from the sun are incident normally. Experiments have proved it that this area is equivalent to area of a circular earth i.e.$\mathbf{A_{earth} = \pi r^2}$ "r" is radius of the earth.

$$P_a = \frac{4.0*10^{26}*(6.4*10^8)^2}{4*(1.5*10^{11})^2} = 1.82*10^{17}W$$

Power emitted by the sun $= \delta A T_e^4$

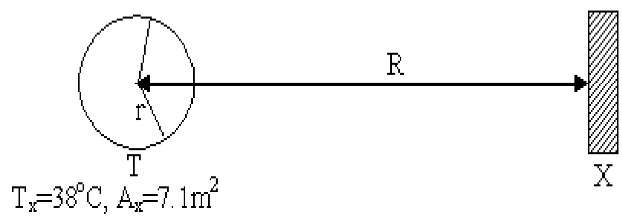
$$= 5.67*10^{-8}*4\pi(6.4*10^6)^2 T_e^4 = 2.92*10^7 T_e^4$$

At equilibrium, the power absorbed by the earth from the sun = power emitted by the earth when it is at T_e.

$$\therefore 1.82*10^{17} = 2.92*10^7 T_e^4 \Rightarrow T_e = 281.0K$$

2. A body of surface area 7.1m^2 at temperature of 38^{o}C absorbs radiation from a spherical blackbody which is at a distance of 5.0×10^3m from its centre and has a radius of 20m.

Find the equilibrium temperature of the blackbody and state the assumption made.
Solution



Power radiated by blackbody = δAT^4

$= 5.67\text{x}10^{-8}\text{x}4\text{x}\pi\text{x }(20)^2T^4 = 2{,}85\text{x}10^{-4}T^4$

Power transmitted by 1m² of the space that surrounds the sphere (the same power is incident normally on 1m² of X)

$= \frac{2.85x10^{-4}xT^4}{4\pi(500)^2} = 9.1\text{x}10^{-13}T^4$

This implies that total power absorbed by X of area 7.1m² is

$9.1\text{x}10^{-13}T^4\text{x}7.1 = 6.44\text{x}10^{-12}T^4$

By Stefan's law, the power radiated by X $= \delta A_x T_x^4$

$P_e = 5.67\text{x}10^{-8}\text{x}7.1\text{x }(311)^4 \approx 3766.0\text{W}$

At equilibrium, $P_a = P_e$ i.e.

$3766.0 = 6.44\text{x}10^{-12}T^4$

T = 4917.6K

The assumption made is that; all radiations from the black body are incident normally on the whole body.

3. The surface temperature of the sun is 6000K and the Earth orbital radius about the sun is $1.5\text{x}10^{11}$m. Calculate the amount of solar energy approaching the Earth from the sun. State any assumption made.
4. Estimate the temperature of the Earth assuming that it is in radiative equilibrium with the sun. (ans:290K)
5. The total power out put of the sun is $4\text{x}10^{26}$W. Given that the mass of the sun is $1.97\text{x}10^{30}$kg and its density is $1.4\text{x}10^3\text{kgm}^{-3}$. Estimate the temperature of the sun.

Solution

Volume of the Sun $= \frac{mass}{density} = \frac{1.97\times10^{30}}{1.4\times10^3} = 1.407\times10^{27}m^3$. Assuming the sun is spherical, then $1.407\times10^{27} = \frac{4}{3}\pi R_S^3 \Longrightarrow R_s = 6.95\times10^8 m.$

Surface area of the sun = $4\pi R_S^2 = 4\times3.14\times(6.95\times10^8)^2 = 6.07\times10^{18}m^2$

Intensity of radiation or solar constant= $\frac{P_s}{A} = \frac{4\text{x}10^{26}}{6.07\times10^{18}} = 6.59 \times 10^7$. Therefore, from $\frac{P_s}{A} = \sigma T_s^4 \Longrightarrow T_s = \left(\frac{6.59\times10^7}{5.67\times10^{-8}}\right)^{\frac{1}{4}} = 5839K$

Exercise

1. The tungsten filament of an electric lamp has a length of 0.5m and a diameter of 6×10^{-5}m. The power rating of the lamp is 60W. Assuming the radiation from the lamp is equivalent is 80% that of a perfect blackbody radiator at the same temperature, estimate the steady temperature of the filament.

2. The intensity of radiant energy from a blackbody is maximum at a wavelength of 1.5×10^{-6}m, find the corresponding temperature of the blackbody.

3. The resistance of a tungsten wire of an electric lamp at 20^oC is 50Ω. At an operating voltage of 240V, the current through the filament is 0.5A. Given that the temperature coefficient of resistance of tungsten is constant and is equal to 5.0×10^{-3}K. Find the;
   i) temperature of the filament at the above operating voltage
   ii) Stefan's constant taking the effective area of the filament to be 0.93cm^2 and assumed a blackbody.

4. The filament of an electric bulb attains a temperature of 1600K when the power supplied to it is 25W.
   i) Find the temperature of the filament if the power supply is increased to 60W.
   ii) Find the length of the filament at 1600K if the diameter is 5.0×10^{-5}m.
   iii) Calculate the difference in the wavelength of the radiation emitted with maximum intensity at the two temperatures (Assume the filament to radiate like a blackbody).

5. A solid copper sphere of diameter 10mm and temperature 150K is placed in an enclosure maintained at temperature of 290K. Calculate, stating the assumptions made, the initial rate of temperature rise of the sphere. ($\rho_{Cu} = 893kg/m^3$, s.h.c of Copper=$310\text{Jkg}^{-1}\text{K}^{-1}$)

6. (i) Assuming the sun is a sphere of radius 7.0×10^8m at temperature of 6000K; estimate the temperature of the surface of mars if its distance from the sun is 2.28×10^{11}m.
   (ii)The energy intensity received by the earth from the sun is 2.4 x 10^3 Wm^{-2}, calculate the surface temperature of the sun and state any assumptions made

7. (a) (i) Define black body and black body radiation.
   (ii)Explain the mechanism of heat transfer through good conductor metals and gases.
   (b) Describe the experiment for determining thermal conductivity of a good conductor.

(c) (i) Explain why when determining thermal conductivity of cork, it is thin.

(ii) Assuming the sun to be a sphere of radius 7.0×10^8m at 6000K. Estimate the temperature of planet mars which is at a distance of 2.28×10^{11}m from the centre of the sun.

9. (a) (i) Explain why when charcoal is steadily heated, it appears reddish before turning white.

(b) (i) Explain what is meant by a black body absorber.

(ii) Describe how the above body can be realized in practice.

(iii) Account for the fact that metals are good conductors of heat.

(c) (i) State the factors which affect the rate of heat flow through a material.

10. (a) (i) State Wien's displacement law and Stefan's law of black body radiation.

(ii) Show how relative intensity of a black body varies with wave length at different temperatures.

(iii) Explain how the curves in a (ii) above account for the change of color by a metal being heated.

b) (i) Define solar constant and Emissivity

(ii) State Prevost's theory.

(c) Use the figure below to find λ_{max} which corresponds to curves with peaks for;

(i) Radiation in the sun's core where temperature is approximated to 15×10^6K

(ii) Radiation in an interstellar space whose temperature is 2.7k.